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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXIV

JUNE 1901

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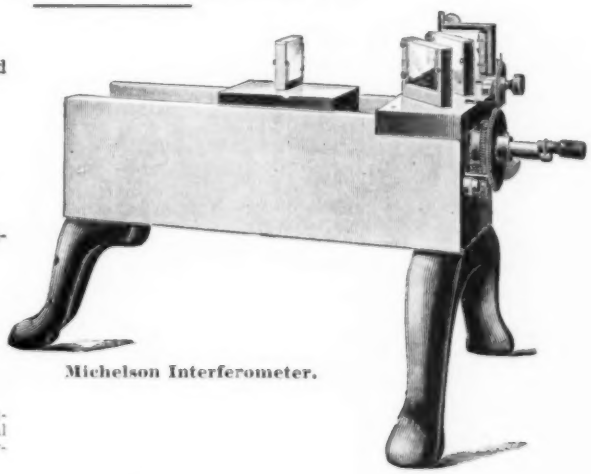

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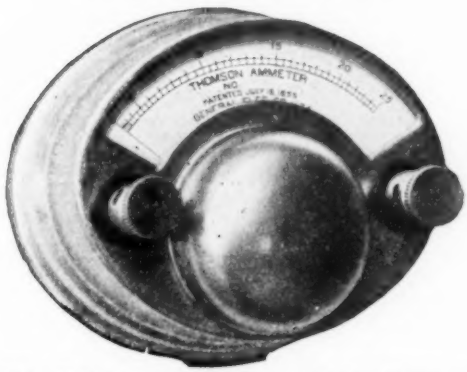

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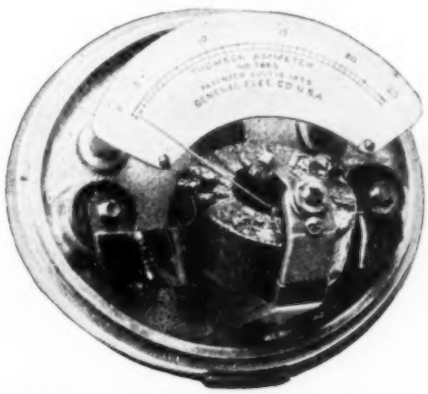

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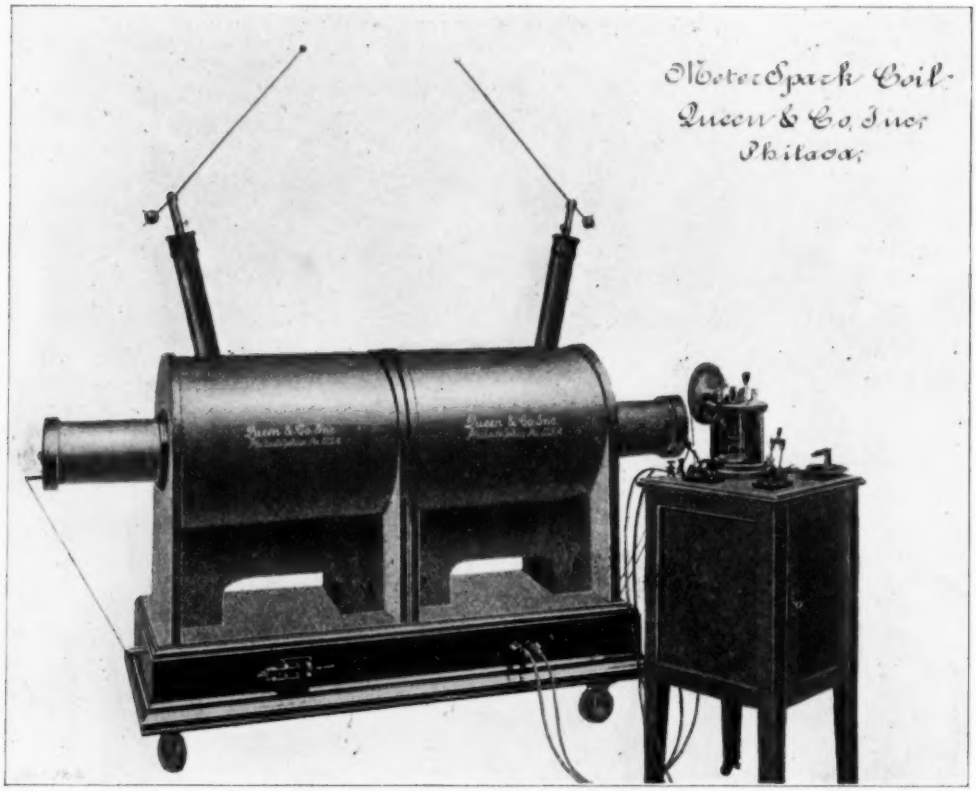

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THE

PHYSICAL REVIEW.

THE INFLUENCE OF TEMPERATURE UPON THE PHOTO-ELECTRIC EFFECT.

By John Zeleny.

1. The discharge of negatively electrified bodies by the impinging of light waves upon them has been studied in various phases by a large number of observers. The results obtained by these have led to the hypothesis [1] that the discharge is due to an ionization produced at the surface of the body by the rapid absorption of certain of the waves of the impinging light. There is but little known, however, as to the relative parts played in the ionization by the material of the body and by the gas occluded in it or existing as a layer upon its surface.

The study of the effects of any cause, which produces variations in the rate of the discharge, is of interest inasmuch as it may aid in leading to a more exact knowledge of the fundamental conditions necessary for the discharge itself. The changes which have been observed in the rate of the discharge by altering either the electric force between the electrodes or the distance between them or the pressure of the gas, have recently been explained by Townsend [2] as due mainly to the secondary effect of the production of new ions in the gas by the rapid passage through it of the negative ions formed by the light at the metal surface. The influence of the temperature of a body upon the rate of the discharge from its surface has not been

[1] J. J. Thomson, The Discharge of Electricity through Gases; E. Rutherford, Proc. Camb. Phil. Soc., Vol. IX., Pt. VIII., 1898.

[2] J. S. Townsend, Phil. Mag., Feb., 1901, p. 220.

systematically studied, although the temperature of a body no doubt has a large effect upon the amount of the gas occluded and upon the surface layer of the gas as well as upon the molecular structure of the body itself. Hoor [1] found that by heating the zinc plate used in his experiments with a Bunsen burner to 55° C., the effect of the light was much diminished. He cites this as a proof that the discharge originates in the surface layer of gas, which he thinks had been driven off at the higher temperature. Stoletow [2] states that he obtained results the opposite to those of Hoor. He placed two platinum electrodes in an air bath and by gradually heating it to above 200° C. he found that the rate of the discharge produced by the light was rather increased, although not regularly.

2. The present paper gives the results of some preliminary experiments on the influence of the temperature of a body upon the rate at which electricity is discharged from its surface by ultra-violet light.

In the first experiment which was tried an open cylinder made of brass wire gauze was placed upon the plate of a gold-leaf electroscope. A coil of platinum wire was supported in the center of the cylinder and connected to earth. The temperature of the platinum wire could be altered by sending an electric current through it, and the light from an arc lamp could be concentrated upon the wire by means of a quartz lens. When the electroscope and the gauze cylinder were charged positively and hence the earthed platinum wire negatively, it was found that the discharge produced by the light was two to three times as rapid while the wire was heated to about redness as when it was at room temperature.

3. The following arrangement was then adopted for studying the effect more closely. The apparatus which was used is represented diagramatically in Fig. 1, the parts, however, not being drawn to scale.

The surface from which the discharging effect of the light was observed was for the most part that of a platinum wire. Its temperature was varied by means of an electric current. A wire about 20 cms. long and having a diameter of .038 cms. was bent into the

[1] M. Hoor, Beibl. z. d. Ann. d. Phys. u. Chem, p. 731, 1889.
[2] A. Stoletow, Comptes Rendus, 108, p. 1241, 1889.

form shown at S in the figure (the total diameter being about 2.5 cms.). Its actual position in the apparatus is represented at P. The ends of the wire were soldered to the ends of the brass rods T and T'. To the same solder junctions were fastened the two copper wires U and U' which were connected to the voltmeter V that was used for measuring the fall of potential between the ends

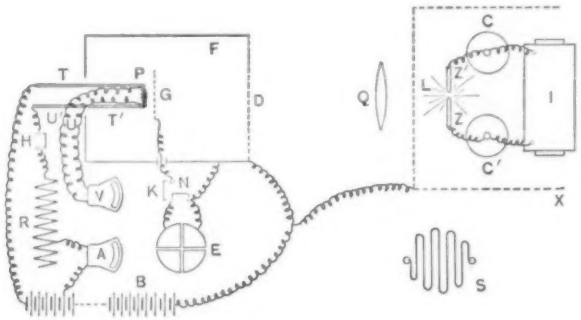


Fig. 1.

of the wire. The platinum wire P was maintained at a definite potential by means of the battery of large storage cells B, the negative end of which was connected to the rod T while the positive end was joined to earth.

By closing the key H, a current from the last few cells of the battery could be passed through the wire P, and its amount was measured by the ammeter A and regulated by means of the changeable resistance R.

The simultaneous readings of the voltmeter V and the ammeter, A determined the resistance of the wire P. From this resistance the average temperature of the wire could be obtained since the resistance of the wire had previously been determined at the melting point of ice and at the boiling points of water and of sulphur. The temperature coefficient of the wire was quite low, which indicates that the platinum was not pure. The average temperature of the wire is no doubt somewhat different from the temperature of the surface of the wire with which we are concerned in these experiments. This is due partly to the uneven distribution of the temperature throughout the cross-section of the wire and partly to the cooling of the wire at the points of attachment.

The source of light used was the electric spark L produced between the two zinc rods Z and Z' by the induction coil I. A six-inch Apps coil operated by the attached hammer was used. The Leyden jars C and C' served to increase the capacity. The source of light L was over 50 cms. from the wire P, and the light was concentrated upon the latter by the quartz lens Q which had a diameter of 9 cms. The visible light was brought to a focus at some distance beyond P so that the ultra-violet light would cover but the central portion of P.

An insulated circular brass wire gauze G, having a diameter of 6 cms., was placed 1.6 cms. in front of the wire P. By means of the key K the gauze could be connected to one pair of quadrants of the electrometer E, the other pair of which was joined to earth. The gauze G could be brought to zero potential by closing both of the keys K and N.

The negative potential of P created the electric field between P and G under the action of which negative ions produced at P were carried to G. The rate of discharge of P could therefore be measured on the electrometer E.

The parts P and G were surrounded by an earthed metallic cage F, 20 cms. long and 16 cms. in diameter. The front part was made of wire gauze for the admission of the light. The electrometer with the keys and connecting wires was also covered by metallic shields not shown in the figure, the keys being manipulated from the outside by means of levers. Lastly the induction coil with the Leyden jars and spark gap were completely enclosed in a wire gauze cage connected to earth.

4. The following was the procedure in taking readings with the apparatus. The proper current was allowed to flow through the platinum wire for about a minute to allow the wire to assume a stationary temperature. Then, for temperatures below those for which the wire showed any luminosity, the zero of the electrometer was read with the key K closed and N open. The induction coil was next started by closing a key not shown in the figure, and a stop watch was started at the same time. The readings on the ammeter and voltmeter were then taken. At the expiration of a minute the induction coil was stopped, and the electrometer reading was taken after it had assumed a stationary value.

When the wire P was hot enough to produce a discharge owing to its temperature, the zero of the electrometer was read with both of the keys K and N closed. The coil was started, and then simultaneously the key N was opened and the stop watch started. The readings of the ammeter and voltmeter were then taken as before. At the end of the minute the key K was opened, disconnecting G from the electrometer, the coil was stopped and the electrometer reading taken when stationary.

A similar procedure was followed for finding the discharge due to the heated wire alone, when unaided by the ultra-violet light.

5. It was found that, while for temperatures not far from that at which the wire begins to emit light the rate of discharge was greater than that at room temperature, for some intermediate temperatures, however, the rate of discharge was less.

This is shown by the following set of readings, taken after the wire had just been used for another set of readings.

TABLE I.

PLATINUM WIRE.

Reference number.	Time of observation.	Resistance of wire.	Temperature.	Electrometer deflection.
1	4:10	No current.	20° C.	146
2	4:13	No current.	20	147
3	4:17	.342	80	132
4	4 20	.348	95	117
5	4:23	.390	170	91
6	4:27	.450	275	108
7	4:30	.588	520	235
8	4:33	.663	655	7*
9	4:35	.662	655	354

The platinum wire was maintained throughout at -70.5 volts.

The reading number 8 was taken without any light on the wire and shows that at that temperature the wire was beginning to discharge negative electricity due to the heat alone. This discharge increases more rapidly with the temperature than does the discharge caused by the light, so that by increasing the temperature less than 100° above the highest used above the two become about equal. While the discharge due to the light alone can be obtained by sub-

tracting from the total effect that caused by the heat alone, it becomes difficult to attain any high accuracy after the discharge caused by the heat becomes large, since small changes in temperature which are liable to occur affect this latter discharge greatly. This gives a limit to the temperatures that can be employed in the experiments.

The results of the above table are shown graphically in Fig. 2, where the temperatures are plotted as abscissas and the electrometer deflections as ordinates.

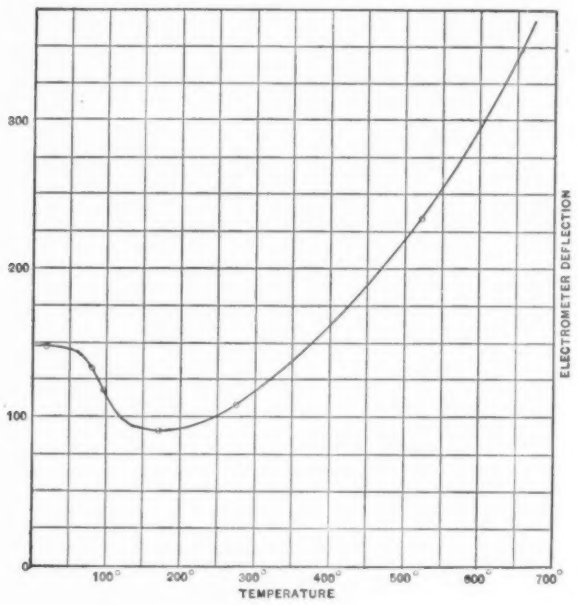


Fig. 2.

It is seen from the curve that the rate of discharge at first diminishes with increase of temperature and then finally increases. The amount of the diminution of the rate of the discharge at the minimum point of the curve, relative to the discharge at room temperature, was quite different in different sets of readings taken in the above manner. It is especially to be noted that when the wire had not been used for some time the first set of readings showed at the minimum point the smallest decrease from the readings at room temperature.

The rate of discharge at any temperature was thus found to vary for different conditions of taking the readings, depending considerably upon the immediate previous history of the wire.

The results indicate that the changes which are produced by an alteration of temperature and which affect the conductivity reach their state of equilibrium but slowly in some cases, and possibly do not attain the same stage for an increasing as for a decreasing temperature.

6. Some of these peculiarities are shown in the following set of readings where one series of observations was taken with the temperatures of the wire increasing step by step to the highest used and then diminishing similarly to room temperature; and a second series was taken like the above except that between the individual readings the wire was allowed to cool to room temperature and a reading always taken for that point.

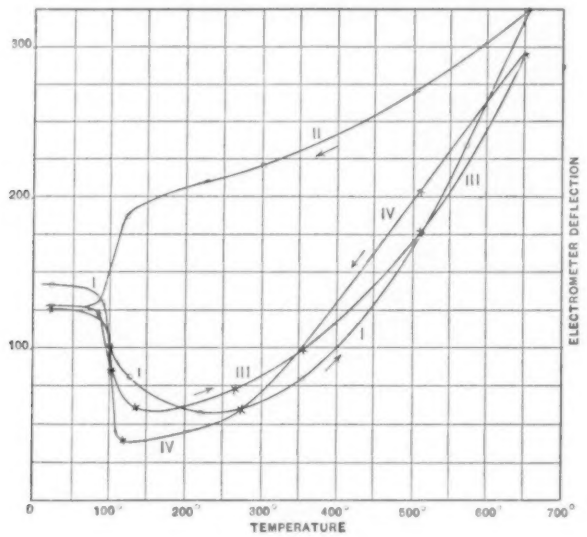


Fig. 3.

The results are shown graphically in Fig. 3.

Curve I. represents the readings for continuously increasing temperatures.

Curve II. represents the corresponding readings for decreasing temperatures.

TABLE II.

PLATINUM WIRE.

Reference number.	Time of observation.	Resistance of wire.	Temperature.	Electrometer deflection.
1	8 : 40	No current	20°C	140
2	8 : 43	No current	20	142
3	8 : 46	.345	90	129
4	8 : 49	.350	100	99.5
5	8 : 52	.371	125	86
6	8 : 55	.420	220	57
7	8 : 58	.492	355	80.5
8	9 : 01	.617	570	234
9	9 : 05	.664	655	8.5*
10	9 : 06	.664	655	332.5
11	9 : 09	.578	505	270
12	9 : 12.5	.464	300	221
13	9 : 16	.368	120	187.5
14	9 : 19	.343	85	130
15	9 : 22	———	20	129.5
16	9 : 25	———	20	128
17	9 : 29	.344	85	123
18	9 : 31	———	20	125
19	9 : 35	.350	100	86
20	9 : 38	———	20	122
21	9 : 41	.376	135	66
22	9 : 44	———	20	132
23	9 : 47	.445	265	72.5
24	9 : 50	———	20	106
25	9 : 53	———	20	111.5
26	9 : 55.5	.493	355	97
27	9 : 58	———	20	89
28	10 : 01	———	20	95
29	10 : 05	.581	510	177
30	10 : 08	———	20	92
31	10 : 11	.663	655	13*
32	10 : 12.5	.663	655	308
33	10 : 16	———	20	112
34	10 : 19	.581	510	203.5
35	10 : 22	———	20	107
36	10 : 25	.453	275	59
37	10 : 27	———	20	104.5
38	10 : 30	.364	115	38
39	10 : 33	.364	115	42
40	10 : 35	———	20	123.5
41	10 : 38	.350	100	101.5
42	10 : 41	———	20	126.5

Readings number 9 and 31 which are marked by a * were taken without the light, the observed discharge being produced by the heated wire alone. The potential of the platinum wire was —71 volts.

Curve III. represents the readings for increasing temperatures when the wire was cooled to room temperature after each reading.

Curve IV. represents the corresponding readings for decreasing temperatures.

The curves I., III. and IV. are practically alike except that IV. descends lower at the minimum point than the others, a peculiarity which appeared in repetitions of the observations.

Curve II. shows a larger conductivity than in the other cases, for all of the higher temperatures; as if on lowering the temperature some change produced at the higher temperature persisted through some of the lower ones.

It is noticed that at about 100° there is the most rapid change in the curves, the upper one (II.) and the lower ones approaching each other rapidly, indicating that at this temperature some more or less sudden change in the conditions takes place.

It is not possible to tell how much variation occurred in the intensity of the light during the progress of the readings, but there is reason to believe that it was not large and certainly not large enough to overshadow the peculiarities observed. The readings at room temperature at the beginning and at the end of the observations are not much different, while the two readings at the highest temperature used in the two series are also of nearly the same value. The ratio of the readings at the highest temperature to those at the temperature of the room is also the same in this series as in others which were taken at different times. Whatever disagreement exists among the readings at room temperature taken between the other readings is not necessarily all due to changes in the source of light, as there is an after effect here also which is more pronounced after a return from certain temperatures than after others.

7. The large difference between the readings at certain points for the same temperature, depending upon whether a lower or a higher temperature has been used immediately preceding is well shown by taking cycles of readings like the following:

The potential of the platinum wire was -72 volts.

It is seen that number 4 which follows the high temperature is about three times as large as number 2, which was taken after the room temperature, although both points are for approximately the same temperature.

TABLE III.

Reference number.	Resistance of wire.	Temperature.	Electrometer deflection.
1	No current.	20° C.	92
2	.417	220	62
3	.623	590	203
4	.439	255	171
5	No current.	20	109

8. In order to find whether the effect was influenced by a change in the electric intensity between the platinum wire and the gauze, the distance between them was reduced to one third of its former value. The potential of the wire was kept the same as before, and the following set of readings was taken :

TABLE IV.

Reference number.	Resistance of wire.	Temperature.	Electrometer deflection.
1	No current.	20° C.	143
2	.419	220	64
3	.636	605	244
4	.428	240	207.5
5	No current.	20	126

Here again the large difference between numbers 2 and 4, taken after the low and high temperatures respectively, is apparent and is of about the same order as before.

9. A change in the intensity of the light was also tried, the spark being made much weaker. The following set of readings taken shows the same peculiarity with respect to numbers 2 and 4 as appears in the other cases :

TABLE V.

Reference number.	Resistance of wire.	Temperature.	Electrometer deflection.
1	No current.	20° C.	39
2	.430	240	17
3	.638	610	64
4	.426	235	43
5	No current.	20	27

10. In the above sets of readings the time interval between the successive observations was about three minutes. It was found that the conductivity observed at certain temperatures after the wire had just been cooled from some higher temperature was much larger than if the wire had not been heated. This increased conductivity is, however, in part, at least, but temporary. Some change is produced by the heating from which the wire recovers (at least partly), but in the time interval used above between the readings the recovery had not been completed, the conditions at the wire not having reached their steady state. This abnormal conductivity, therefore, gradually diminishes with time. The following set of readings illustrates this point:

TABLE VI.

Reference number.	Time of observation.	Resistance of wire.	Temperature.	Electrometer deflection.
1	9 : 22	No current.	20° C.	104
2	9 : 26.5	——	20	112
3	9 : 30	.436	250	65
4	9 : 33	.435	250	59
5	9 : 35.5	.435	250	58
6	9 : 39.5	.631	605	181
7	9 : 43	.633	605	181
8	9 : 46	.435	250	156
9	9 : 49	.434	250	138
10	9 : 53	.434	250	123
11	9 : 56	.433	245	120
12	10 : 01	.433	245	119
13	10 : 05.5	——	20	122
14	10 : 09	——	20	118.5
15	10 : 13	——	20	118.5
16	10 : 17	.434	250	62
17	10 : 20.5	.434	250	58.5
18	10 : 24	.434	250	52.5
19	10 : 27.5	.434	250	54.5
20	10 : 32	.626	590	5.5*
21	10 : 34	.626	590	167
22	10 : 36.5	.445	265	138.5
23	10 : 40	.445	265	120.5
24	10 : 43.5	.446	265	109.5
25	10 : 46.5	.446	265	102.5
26	10 : 50	——	20	106.5
27	10 : 54	.614	570	158.5
28	11 : 03	.445	265	102.5
29	11 : 06	——	20	103.5

Reading number 20 marked by a * was taken without the light. The readings from number 8 to number 12 show that after the wire had been reduced from over 600° to about 250° the conductivity at first diminished quite rapidly with time, but after fifteen minutes the change was slow, its value being then about the same as at room temperature. The readings from number 22 to number 25 also illustrate this same point. In these sets of readings the wire was exposed to the light a number of times after the heating, which fact might have been the cause of the change which produced the diminution in the conductivity. That this was not the case is shown by readings 26 to 29. Here number 28 was taken, without intermediate readings, nine minutes after number 27, and it is seen that the value of the conductivity is diminished at once to approximately the same as at room temperature.

After changing the temperature of the wire from the room temperature to about 250° the readings 3 to 5 and the readings 16 to 19 show that here also there was some diminution of the conductivity with time.

11. The preceding results show that for the range of temperatures used the conductivity produced by the impinging of the ultraviolet light upon a negatively charged platinum wire, at first diminishes with the temperature, reaches a minimum value and then increases.

After reducing the temperature of the wire to certain intermediate temperatures, the conductivity is larger at first than if such temperatures had not been exceeded shortly before; the wire acquiring its steady state but slowly in such cases. Whether the steady state is the same for a descending as for an ascending temperature is not yet determined. Careful experiments will be made on this point, for if the final state is not the same in the two cases, this must be taken as good evidence that the result is due to a changed molecular arrangement of the wire.

The recovery of the steady state after a reduction of temperature is more rapid at certain temperatures than at others.

The temperature at which, upon descending to it, the recovery begins to be especially rapid is in the neighborhood of about 100° C., at which place also the conductivity diminishes quite rapidly in the

case where readings are taken when the temperatures are being increased.

12. Some experiments were performed in which the platinum wire used previously was replaced by an iron wire bent in the same form.

TABLE VII.

IRON WIRE.

Reference number.	Time of Observation.	Resistance of wire.	Electrometer deflection.
1	9 : 16	No current.	2.
2	9 : 21	.35	1.25
3	9 : 24	——	2.75
4	9 : 26.5	.35	1.75
5	9 : 30	——	2.75
6	9 : 33	.558	1.25
7	9 : 37	.558	1.
8	9 : 39	——	2.75
9	9 : 42	.714	1.25
10	9 : 46	——	1.75
11	9 : 49	.987	4.75
12	9 : 52	——	3.25
13	9 : 59	1.19	8.
14	10 : 01	——	1.5
15	10 : 04	1.30	6.
16	10 : 08	——	2.5
17	10 : 11	1.355	9.75
18	10 : 14	——	2.25
19	10 : 17	1.503	3. *
20	10 : 18	1.503	17.
21	10 : 21	——	2.
22	10 : 24	1.69	26. *
23	10 : 26	1.69	52.
24	10 : 29	——	1.75
25	10 : 33	1.875	162. *
26	10 : 36	1.875	220.
27	10 : 41	——	1.75
28	10 : 47	1.555	3.75*
29	10 : 49	1.555	13.
30	10 : 51	——	1.75
31	10 : 55	1.14	3.25
32	10 : 57	——	2.25
33	11 : 00	.403	1.
34	11 : 03	——	2.25

The conductivity obtained at the lower temperatures was very small and approached the lower limit of the electrometer.

The preceeding is a set of readings taken with the iron wire at −70 volts, the current through it being broken after every reading and the conductivity at room temperature measured. The resistance calculated from the voltmeter and ammeter readings is given in each case as before. The corresponding temperatures cannot be given because the constants of the wire were not determined. The resistance of the wire at 25° C. was .31 ohm, and when it was just beginning to glow on account of the current sent through it its resistance was about 1.30 ohms.

The readings marked by a * were taken without the light, the observed conductivity being caused by the heated wire alone.

Before reading number 13 the wire was heated more than was desired, so it was cooled to and allowed to remain at room temperature for some time. In all the readings where the resistance is not given the wire was at room temperature.

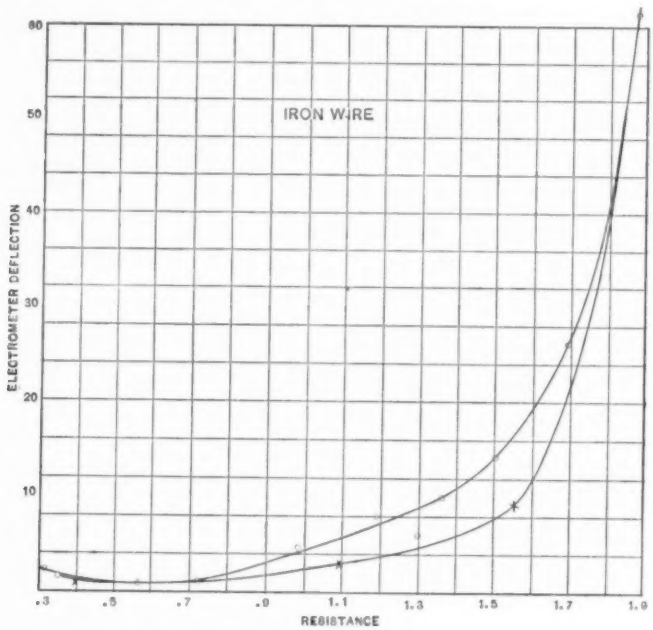


Fig. 4.

The results are shown graphically in Fig. 4, from which it is seen that as the temperature gets above what would correspond to about 700° C. the conductivity increases very rapidly. This in-

crease with the temperature is proportionally much larger than was the case with the platinum wire. It may be worthy of notice that the large change begins in the neighborhood of the point of recalescence of the iron.

As the temperature was increased above that of the room for a considerable range the observed conductivity was diminished as was the case with the platinum wire. The smallness of the readings at these temperatures may put some doubt upon the reality of this diminution ; but a number of other readings in this same region all gave a like result. It is possible that the difference between the ascending and the descending curves is all due to variations in the light.

A set of readings taken with continuously increasing and then decreasing temperatures, as was done in the case of the platinum, did not show the same tendency for the readings taken when the temperatures were decreasing, to exceed those taken for the same points when the temperatures were increasing.

13. The photo-electric effect presents the peculiarity that negative charges alone are dissipated by the light. It may be, from the very nature of the action, that only negative charges can be affected ; but it is possible that the existing forces simply hinder the removal of the positive charges more than of the negative and that in the former case the action of the ultra-violet light cannot overcome them.

Now when a body is heated it acquires at a certain temperature the power to discharge positive electricity from its surface, and at a somewhat higher temperature it discharges the negative also. It seemed possible therefore that, especially if the nature of the process of ionization due to the heat is at all analogous to that in the discharge by ultra-violet light, the resistance to the removal of the positive charge by the light might be sufficiently weakened at the higher temperatures to enable the light to overcome it.

Under such conditions if the wire were heated to a temperature where it was almost on the point of discharging positive electricity without any light, the addition of the influence of the latter ought to produce a discharge ; while on the other hand if the wire were sufficiently hot to produce a discharge of the positive, the addition of the light ought to increase this discharge.

The experiment was tried both with a platinum and an iron wire in the apparatus, charged in each case to +75 volts and maintained at a temperature not quite sufficient to discharge positive electricity. When the light was turned on, no certain leakage could be detected and if present at all it was certainly less than one division per minute.

When the platinum wire was heated to a dull red so that it discharged positive electricity slowly, the following readings were taken without the light and with the light, the action being allowed to continue for three minutes in each case.

TABLE VIII.

PLATINUM WIRE. POSITIVE.

Time in minutes.	Electrometer deflection.	
	Without Light.	With Light.
1	14.	14.5
2	27.	26.5
3	39.5	39.5

An iron wire heated similarly gave the following readings per minute, the light being on and off alternately in this case.

TABLE IX.

IRON WIRE. POSITIVE.

Light.	Electrometer deflection.
On.	102.5
Off.	101.
On.	101.5
Off.	101.
On.	101.

The results indicate that within the limits of the experiments the action of the light was without appreciable effect.

The process of the ionization in the discharge due to the heat must be quite different from what it is in the case of the light discharge. Such mechanical movements as may be produced by the absorption of the light do not appreciably affect the discharge of the positive electricity which is caused by the heat. Whether such

movements have any influence upon the discharge of the negative electricity by the heat is not certain from the experiments performed.

The following simple view may be taken of the cause of the unipolarity of the discharge produced by ultra-violet light.

The absorption of the light waves gives rise to intra-molecular forces which tend to separate a small negatively charged corpuscle[1] from the rest of the neutral molecule. The maximum impulse so given to the negative particle may give it a velocity away from the surface sufficient to carry it beyond what is called the range of molecular action, while the same outward impulse acting upon the much larger positive mass is not sufficient to get it beyond the influence of the attracting forces. We must consider the attraction tending to bring the departing particle back to the surface as not alone an electrical one or due altogether to the part of the molecule from which it has been separated, but as arising also from the attraction of all of the molecules within the range of molecular action. This latter force pulling the particle back to the surface is proportional to its mass and is therefore much the larger for the positive particles. Since the maximum outward impulses, in some way due to the light, are the same when given to a negative particle as when given to a positive one, it is possible to see how the light is effective in discharging the negatively electrified bodies alone.

When an unelectrified body is exposed to the light some of the negative ions being projected from the surface by the light action, diffuse away leaving the body charged positively. The electric field which is thus produced finally puts a stop to the increase of the charge by preventing further diffusion. By blowing past the surface the ions are carried away mechanically in opposition to the electrical forces and so the body may be charged to a higher potential.

To account for the lack of aid given by the light to the discharge of a positively electrified glowing wire we must consider any impulse given to the positive ion by the action of the light as too small to be noticed. This is quite plausible, since, according to the results of J. J. Thomson (loc. cit.), the mass of the negative par-

[1] The existence of such corpuscles is evidenced by the researches of J. J. Thomson. Phil. Mag., December, 1899.

ticle which the light is just able to force away is only about one thousandth part of the mass of a hydrogen atom.

14. The experiments here described were interrupted nearly a year ago by derangements caused by the removal of our laboratory to temporary quarters. As there must still be some delay in resuming the work it was thought best to publish this account at this time.

As soon as possible, however, the work will be continued and it is intended to study carefully the effect of temperature changes upon the discharge from different metals when surrounded by different gases at different pressures. It is also desirable to carry the experiments to higher temperatures. At least two methods suggest themselves.

As stated before the difficulty lies in the presence of the larger conductivity caused by the glowing wire.

Now the results of Rutherford [1] and McClelland [2] show that the ions in the case of discharge by ultra-violet light travel with a greater velocity in the same electric field than those given off by the glowing wire. It would therefore be possible by blowing a properly regulated stream of gas across the path of the ions, to prevent the slower moving ions formed by the glowing wire from reaching the positive electrode, while some of those caused by the ultra-violet light would still get to it. The ions due to the heated wire being thus blown away, it would be possible to make measurements on the changing quantity of those caused by the light.

Probably a better method would be a modification of one devised by Rutherford (loc. cit.) for measuring the velocity of the ions. An alternating electromotive force is used between the two electrodes so that the direction of motion of the ions is periodically reversed. The electric intensity could be so regulated that the slower moving ions due to the heat in making their excursions from the glowing wire would not reach the other electrode, while the negative ions due to the ultra-violet light would reach it, before the reversal of the electromotive force occurred. Here again an electrometer joined to the latter electrode would show effects due alone to the ions produced by the action of the light.

[1] Rutherford, Proc. Camb. Phil. Soc., Vol. IX., Pt. VIII., p. 401.
[2] J. A. McClelland, Phil. Mag., July, 1898.

15. The results of the experiments described above show that temperature has a large influence upon the photo-electric effect. From a platinum wire the negative electricity is at first discharged more slowly as the temperature is increased; the rate of discharge reaches a minimum, however, and then increases as far as the highest temperature used (about 700° C.).

The rate of the discharge produced by the light also depends upon the immediate previous history of the wire. On coming to certain temperatures from a higher one the rate of discharge is much greater than if a lower temperature had been used just before. For small ranges at least this effect appears to be independent of the electric intensity and of the intensity of the light. After the reduction of temperature the rate of discharge gradually becomes smaller with time, showing that the large conductivity at first is due partly at least to the fact that the wire reaches its steady state but slowly. The recovery is much faster at temperatures below a certain point than for those above, the change being fairly abrupt.

From an iron wire the rate of discharge also appears to diminish at first with the temperature but after reaching a minimum it finally becomes a great many times its value at room temperature.

Heating the wire does not change the inactivity of the light for discharging positive electricity, even though the wire is raised to a temperature where, due to the heat alone, the positive electricity is being discharged.

The fact that after the platinum wire has had its temperature reduced the larger conductivity of the higher temperature persists for a time, there being a gradual recovery, shows that at least most of the variations which have been noticed in the experiments described are due to changes taking place at the metal surface and not in the gas between the electrodes. The comparatively abrupt change in this effect occurring at about 100°C. may indicate some molecular alteration of the metal. Any further discussion had best be delayed until the completion of further experiments.

UNIVERSITY OF MINNESOTA, April, 1901.

CURRENTS AND POTENTIALS ON SUBMARINE CABLES PRODUCED BY SINE-WAVE ELECTROMOTIVE FORCES.

BY A. C. CREHORE.

THE problem of the transmission of current and potential in submarine cables has received the attention of the most able physicists, and its solution in a very general form is well established. Fourier gave mathematical solutions in several different forms of the differential equations involved, and these have been applied to the physical problem of the transmission of electrical energy along wires.

While these general solutions are most important as they reveal the nature of the problem suggesting certain relations and laws, which are otherwise not obtainable, yet there is something unsatisfactory about them unless some application is made to a particular case of existing or possible physical conditions, for the actual physical constants involved may give unexpected prominence to certain terms of the general solution, or the physical conditions may be such as to render the solution imaginary.

At the present time there are few examples obtainable of curves of current and potential calculated for particular submarine cables. The case which Lord Kelvin has published with the well-known curves of current and potential, applies only where a battery or constant source of electromotive force is employed. Since condensers are almost universally employed at each end of a cable, the case of the application of a constant potential to the cable itself is not one which exists in practice. With either a battery or a sine-wave transmitter the potential at the transmitting end of the cable itself does not remain constant through a single signal. With a battery transmitter the potential on the cable rises suddenly (almost instantaneously) at the beginning of the signal to approxi-

mately the full battery pressure, and gradually decreases from that point by an exponential curve as the condensers become charged until the cable is connected to earth at the end of the signal. With a sine-wave transmitter the change of potential is less sudden, and the curve of variation follows an entirely different law.

In view of the difficulty attending the exact solution of this problem for an actual case of duplex working, a method of obtaining an approximate solution which gives results not very far from the truth, when a sine-wave form of electromotive force is used in transmission, is given and curves calculated for an existing trans-Atlantic cable. The method only applies to the transmitting end of the cable. The receiving end is perhaps the more important, for the signals must be received in legible shape, if the transmitter has any commercial value; but the transmitting end is also important for it is there that the cable experiences the greatest potentials, and the insulating material the greatest strain. It is important to accomplish the best results at the receiving end by that method which subjects the transmitting end of the cable to the least electrical strain. In a cable the pressure is rapidly reduced in proceeding from the transmitting end, being in the particular Atlantic cable referred to, one-half its value at about 175 knots, the total length of the cable being 2,164 knots.

It has been shown [1] that the measurements of alternating currents and potentials at the transmitting end of the cable referred to are in agreement with the accepted theory within the limits of error of the measurements. No mention is there made, however, of the changes which take place when the electromotive force is first introduced, before the alternating current has attained its final value, and it is the present purpose to study these changes as well as the forms when rectified pulses, consisting of semi-sinuses in the same direction, are employed as in sending certain cable signals.

The curves have more value, however, than would be the case if they were merely approximate solutions for the cable circuits, for they are the exact solutions of the phenomena taking place in a real circuit, which is taken to be the approximate equivalent of the cable circuits.

[1] "A Practical Transmitter Using the Sine-wave for Cable Telegraphy, etc." Trans. A. I. E. E., May, 1900.

The arrangement of the cable circuits is shown in Fig. 1, where the cable terminal is represented at C and the artificial cable at C′. The branches from the apex A to C and C′ contain the bridge condensers. The branch AE from apex to earth contains the armature of the alternating current generator.

The data of the circuits is as follows :

Cable :—No. 3, Atlantic cable of the Commercial Cable Co., from Canso, Nova Scotia, to Waterville, Ireland.

Length,	2,164 knots.
Resistance,	4,895 ohms.
Capacity,	914 microfarads.
Duplex bridge condensers,	80 microfarads each.
Armature resistance,	59 ohms.
Armature inductance,	3.555 henrys.

The equivalent circuit is one in which the same harmonic electromotive force produces the same harmonic current both in magnitude and phase relation to the electromotive force. If the circuits of Fig. 1 from the point A, including both the real and artificial

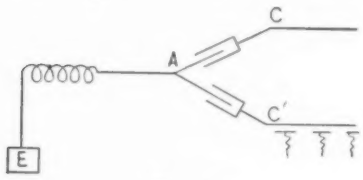


Fig. 1.

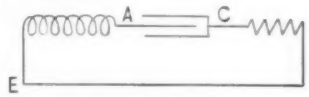


Fig. 2.

cables and the condensers, are replaced by a simple circuit having resistance and capacity, as in Fig. 2, the values of the condenser and resistance may be so proportioned that neither the harmonic current through the armature, nor the phase relation to the electromotive force will be altered.

Since it has been shown that the apparent impedance of the cable differs according to the frequency, the following calculations have been based upon a frequency of $n = 5$. This represents a speed of cable working of 162 letters per minute.

Data of equivalent circuit for frequency $n = 5$.

$$R = 146.255 \text{ ohms.}$$
$$C = 92.208 \text{ microfarads.}$$

Including the resistance and inductance of the armature the data for the whole circuit (Fig. 2) is :

$R = 205.255$ ohms.
$L = 3.555$ henrys.
$C = 92.208$ microfarads.

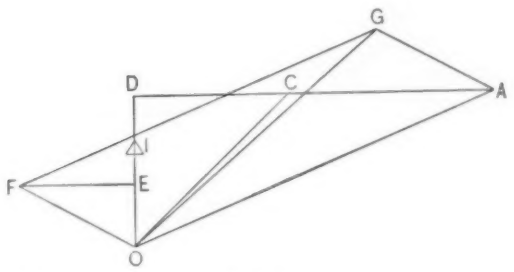


Fig. 3.

All numbers represent maximum harmonic values.

OC = 42.426 volts ; assumed cable potential between C and E, Fig. 1.
OD = 30 volts ; component of cable pressure in phase with the current.
DC = 30 volts ; component of cable pressure in quadrature with the current.
COD = 45°, or $\pi/4$ radians.
OI = .2051 ampère ; current in the apex A, Fig. 1, equal to twice the cable current and in phase with it.
CA = 40.807 volts ; pressure across the bridge condenser, 80 microfarads in branch AC, Fig. 1.
OA = 76.902 volts upon apex between A and E, Fig. 1.
OE = 12.102 volts ; drop through armature resistance.
EF = 22.909 volts ; drop through armature inductance.
OG = 63.773 volts ; generated by the alternator.
DOG = 48° 41′ 10″ or = .8497 radians = θ.

When an harmonic current is flowing in the circuits, Fig. 1, the diagram, Fig. 3, represents the harmonic currents and potentials for the case where the potential on the cable is assumed to be 30 virtual, or 42.426 maximum, volts. The volts required from the generator to produce this upon the cable are, OG, 63.773 (max.) and this produces a maximum of 76.902 upon the apex A. The current in the generator has a maximum value of .2051 ampère and is in advance of the generated electromotive force OG, by an angle of 48° 41′ 10″. The angle between the apex electromotive force and the current AOD is 67° 2′ 20″.

The diagram, Fig. 3, gives the state of electromotive forces when the stable condition is attained. The approximate potentials when the electromotive force is introduced into the circuit will be found by calculating the current which the electromotive force would produce in the equivalent circuit, and assuming that the current in the armature will be approximately the same with the cable circuit. Figs. 4–9 inclusive apply only to the equivalent circuit, and 10 and 11 apply to the cable circuit, Fig. 1. The quantities may be easily calculated for the equivalent circuit. The equation of the impressed electromotive force is

$$e = E \sin \omega t. \tag{1}$$

The differential equation applying to the circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E \sin \omega t. \tag{2}$$

where q is the charge in the condenser.

The solution of this equation is written in three forms as R^2C is greater than, equal to, or less than $4L$. In the case under consideration $4L$ is greater than R^2C, the circuit being oscillatory, and the solution is :

$$q = -\frac{I}{\omega}\cos(\omega t + \theta) + Ae^{-\frac{Rt}{2L}}\sin(at + \varphi), \tag{3}$$

where,

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \quad \text{and,} \quad \theta = \arctan\left(\frac{1}{CR\omega} - \frac{L\omega}{R}\right),$$

$$a = \frac{\sqrt{4LC - R^2C^2}}{2LC} \quad \text{and,} \quad \delta = \arctan -\frac{2La}{R}.$$

A and φ are the arbitrary constants determined by the initial conditions assumed.

For the curves shown it is assumed that the electromotive force is introduced at its zero phase, that is when it has zero value when changing from negative to positive, and that the charge and current in the circuit at the time of introducing the electromotive force are q_0 and i_0 respectively. The equation of the current obtained by differentiating (3) is :

$$(4) \qquad i = I \sin(\omega t + \theta) + \frac{A}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin(at + \varphi + \delta).$$

When $t = 0$, we have $q = q_n$, and $i = i_n$. Hence, from (3),

$$(5) \qquad q_n = -\frac{I}{\omega} \cos\theta + A_n \sin\varphi_n,$$

from (4),

$$(6) \qquad i_n = +I \sin\theta + \frac{A_n}{\sqrt{LC}} \sin(\varphi_n + \delta).$$

Solving for φ_n we have,

$$(7) \qquad \cot\varphi_n = \frac{i_n - I\sin\theta}{\left(q_n - \frac{I}{\omega}\cos\theta\right)a} - \cot\delta.$$

The value of A may be found by substituting (7) in (5) or (6). In calculating numerical values care must be taken to employ the correct value of the angle δ. The tangent of this angle has been given as negative, but this will permit of two angles, in the second or the fourth quadrant. From a consideration of the differential equation [1] (2) it will appear that in order to satisfy the equation, $\cos\delta$ is negative, which determines that δ is an angle in the second quadrant.

The numerical values of the constants for the case assumed are:

[1]
$$L\frac{d^2q}{dt^2} = L\omega I\cos(\omega t + \theta) + \frac{A}{C}\varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi + 2\delta)$$

$$R\frac{dq}{dt} = RI\sin(\omega t + \theta) + \frac{RA}{\sqrt{LC}}\varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi + \delta)$$

$$\frac{q}{C} = -\frac{I}{C\omega}\cos(\omega t + \theta) + \frac{A}{C}\varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi)$$

By addition

$$e = E\sin\omega t = I\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}\sin\omega t + A\varepsilon^{-\frac{Rt}{2L}}\left\{\frac{1}{C}\sin(at + \phi + 2\delta) + \frac{R}{\sqrt{LC}}\sin(at + \phi + \delta) + \frac{1}{C}\sin(at + \phi)\right\}.$$

Hence

$$\frac{1}{C}\sin(at + \phi + 2\delta) + \frac{R}{\sqrt{LC}}\sin(at + \phi + \delta) + \frac{1}{C}\sin(at + \phi) = 0.$$

Hence

$$\cos\delta = -\frac{RC}{2\sqrt{LC}}.$$

$a = 47.088.$
$\delta = 121° 30' 40'' = 2.1208$ radians.
$R/2L = 28.868.$
$T = .1336$ second = period of circuit.

A description of the curves giving the numerical equations in each case is added. It is to be noticed in general that the potential in no case rises higher during the initial stages than the value which it attains when it has reached the steady state, and it thus appears to be a safe rule to find the maximum potentials by the simple calculation of the maximum values when an harmonic condition is reached.

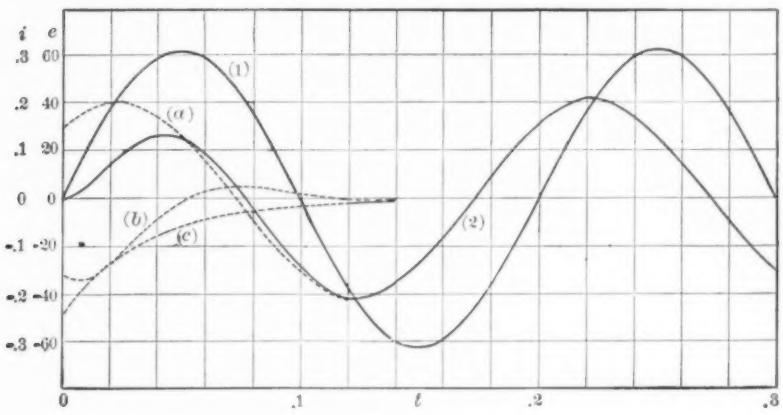


Fig. 4.

Curve (1). Impressed electromotive force, alternating, harmonic, introduced at zero phase, initial charge and current being zero.

$$e = E \sin \omega t.$$
$$e = 63.773 \sin 31.416\, t.$$

Curve (2). Resulting current in equivalent circuit, Fig. 2.
curve (2) = curve (a) + curve (b).

(a) $$i_a = I \sin (\omega t + \theta).$$
$$i_a = .2051 \sin (31.416t + .8497).$$

(b) $$i_b = \frac{A}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin (at + \phi + \delta).$$

$$i_b = -.2406\, \varepsilon^{-28.87t} \sin (47.088t + .6949).$$

(c) $$i_c = -.2406\, \varepsilon^{-28.87t}.$$

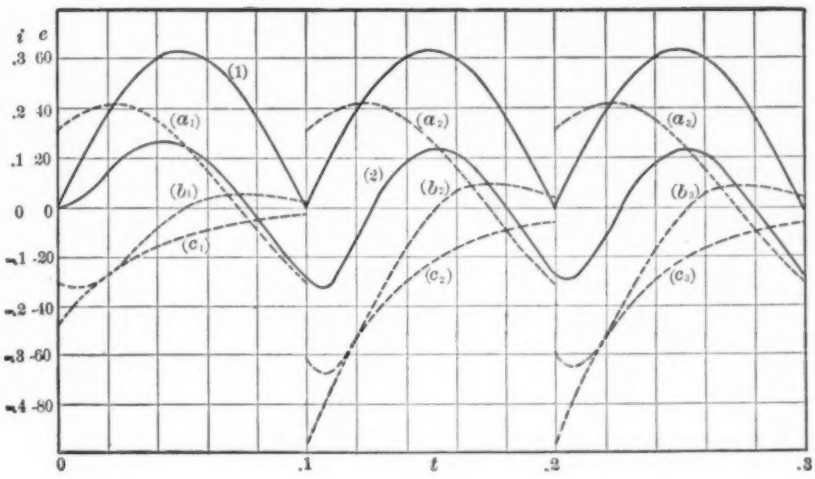


Fig. 5.

Curve (1). Impressed electromotive force, pulsating harmonic, introduced at zero phase, initial charge and current being zero at beginning of first semi-cycle.

Equation the same as for Curve (1), Fig. 4, the origin being transferred to points $t = .1, t = .2, \cdots, t = \frac{n-1}{10}$ for the second, third and n^{th} semi-cycles.

Curve (2). Resulting current in equivalent circuit, Fig. 2.

$$\text{Curve } (2) = \text{curve } (a) + \text{curve } (b).$$

(a_1) Same as (a), Fig. 4, (first semi-cycle). Equation of (a_2) $(a_3), \cdots, (a_n)$ for second, third and succeeding semi-cycles same as (a), Fig. 4, with origin transferred to points $t = .1, t = .2, \cdots, t = \frac{n-1}{10}$ respectively.

(b_1) Same as (b), Fig. 4 (first semi-cycle).

(b_2) $$i_{b_2} = \frac{A_2}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi_2 + \delta).$$

$$i_{b_2} = -.4813\, \varepsilon^{-28.87t} \sin(47.088t + .6669).$$

(b_3) $$i_{b_3} = \frac{A_3}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi_3 + \delta).$$

$$i_{b_3} = -.4811\, \varepsilon^{-28.87t} \sin(47.088t + .6393).$$

(c_1) Same as (c), Fig. 4.

(c_2) $$i_{c_2} = -.4813\, \varepsilon^{-28.87t}.$$

(c_3) $$i_{c_3} = -.4811\, \varepsilon^{-28.87t}.$$

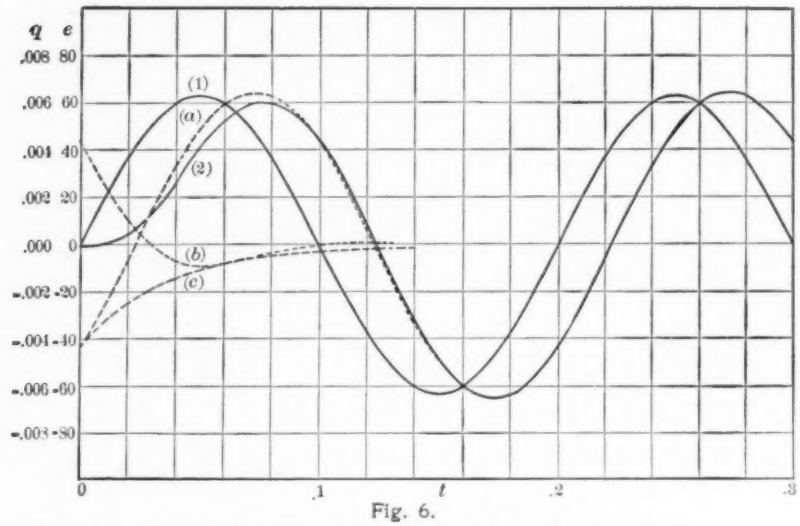


Fig. 6.

Curve (1). Impressed electromotive force alternating. Same as (1), Fig. 4.

Curve (2). Resulting charge in the condenser of circuit, Fig. 2.

Curve (2) = curve (a) + curve (b).

(a) $$q_a = -\frac{I}{\omega}\cos(\omega t + \theta).$$

$$q_a = -.006529\cos(31.416t + .8497).$$

(b) $$q_b = A\varepsilon^{-\frac{Rt}{2L}}\sin(at + \phi).$$

$$q_b = -.004356\varepsilon^{-28.87t}\sin(47.088t - 1.426).$$

(c) $$q_c = -.004356\varepsilon^{-28.87t}.$$

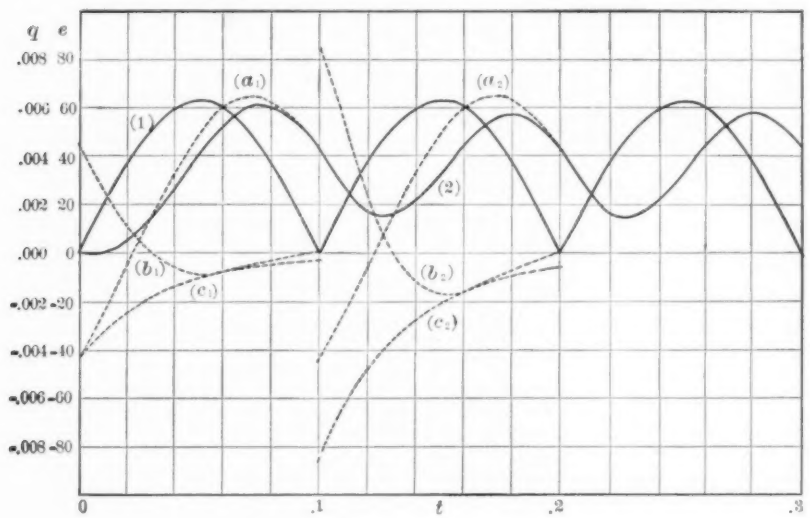


Fig. 7.

Curve (1). Impressed electromotive force pulsating. Same as (1), Fig. 5.

Curve (2). Resulting charge of the condenser.

Curve (2) = curve (a) + curve (b)

(a_1) Same as (a), Fig. 6 (first semi-cycle). Equation of (a_2), (a_3), ..., (a_n) for second, third and succeeding semi-cycles are the same as (a), Fig. 6, with the origin transferred to points $t = .1, t = .2, \ldots, t = \frac{n-1}{10}$.

(b_1) Same as (b), Fig. 6.

(b_2) $q_{b_2} = A_2 \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi_2)$.

$q_{b_2} = -.008714\varepsilon^{-28.87t} \sin(47.088t - 1.4539)$.

(b_3) $q_{b_3} = A_3 \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi_3)$.

$q_{b_3} = -.008710\varepsilon^{-28.87t} \sin(47.088t - 1.4815)$.

(c_1) Same as (c) Fig. 6.

(c_2) $q_{c_2} = -.008714\varepsilon^{-28.87t}$.

(c_3) $q_{c_3} = -.008710\varepsilon^{-28.87t}$.

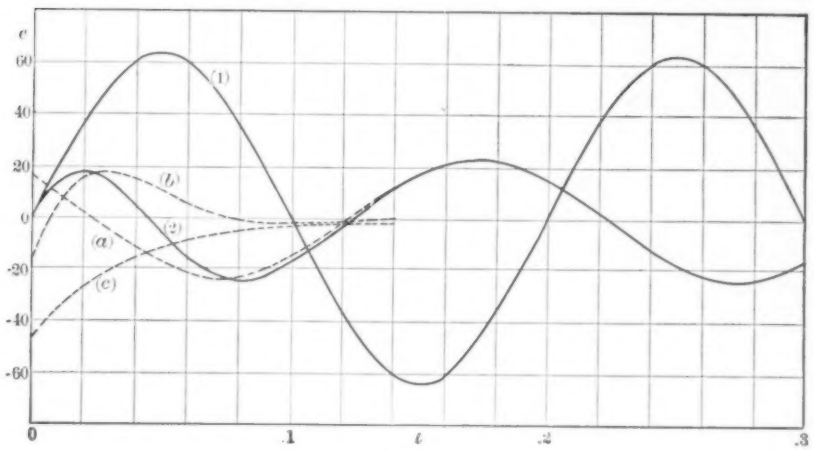


Fig. 8.

Curve (1). Impressed electromotive force alternating. Same as (1), Fig. 4.

Curve (2). Resulting electromotive force required to overcome the inductance of the armature in circuit, equal to $L\frac{d^2q}{dt^2}$.

Curve (2) = curve (a) + curve (b).

(a) $\left(L\frac{d^2q}{dt^2}\right)_a = L\omega I \cos(\omega t + \theta)$.

" $= 22.909 \cos(31.416t + .8497)$.

(b) $\left(L\frac{d^2q}{dt^2}\right)_b = \frac{A}{C}\varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi + 2\delta)$.

" $= -47.24\varepsilon^{-28.87t} \sin(47.088t + 2.8157)$.

(c) $\left(L\frac{d^2q}{dt^2}\right)_c = -47.24\varepsilon^{-28.87t}$.

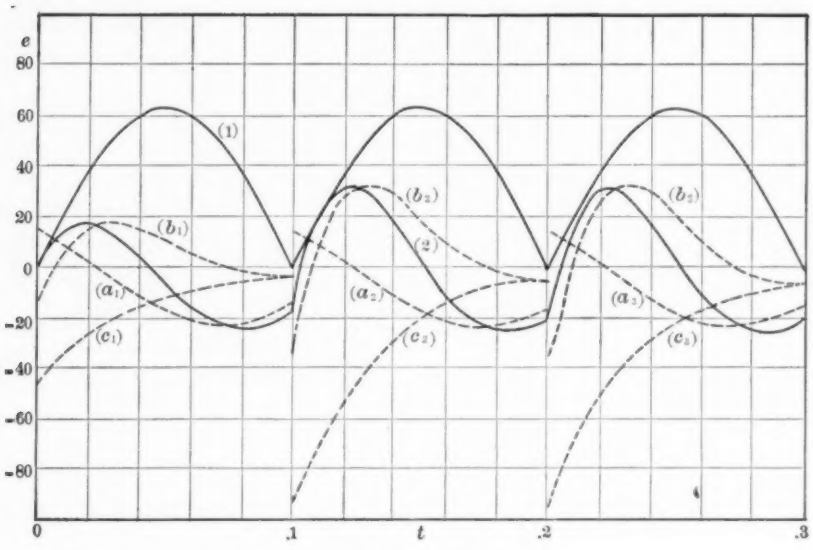


Fig. 9.

Curve (1). Impressed electromotive force, pulsating. Same as (1), Fig. 5.

Curve (2). Resulting electromotive force to overcome the inductance of the armature, equal to $L\frac{d^2q}{dt^2}$.

$$\text{Curve } (2) = \text{curve } (a) + \text{curve } (b).$$

The equations of (a_1), (a_2), ..., (a_n) for the first, second and following semi-cycles are the same as (a), Fig. 8, with the origin at the points $t = 0$, $t = .1$, ..., $t = \frac{n-1}{10}$ respectively.

(b_1) Same as (b), Fig. 8. (First semi-cycle.)

(b_2) $$\left(L\frac{d^2q}{dt^2}\right)_{b_2} = \frac{A_2}{C}\varepsilon^{-\frac{Rt}{2L}}\sin(at+\phi_2+2\delta).$$

" $$= -94.504\varepsilon^{-28.87t}\sin(47.088t+2.7877).$$

(b_3) $$\left(L\frac{d^2q}{dt^2}\right)_{b_3} = \frac{A_3}{C}\varepsilon^{-\frac{Rt}{2L}}\sin(at+\phi_3+2\delta).$$

" $$= -94.460\varepsilon^{-28.87t}\sin(47.088t+2.760).$$

(c_1) Same as (c), Fig. 8.

(c_2) $$\left(L\frac{d^2q}{dt^2}\right)_{c_2} = -94.504\varepsilon^{-28.87t}.$$

(c_3) $$\left(L\frac{d^2q}{dt^2}\right)_{c_3} = -94.460\varepsilon^{-28.87t}.$$

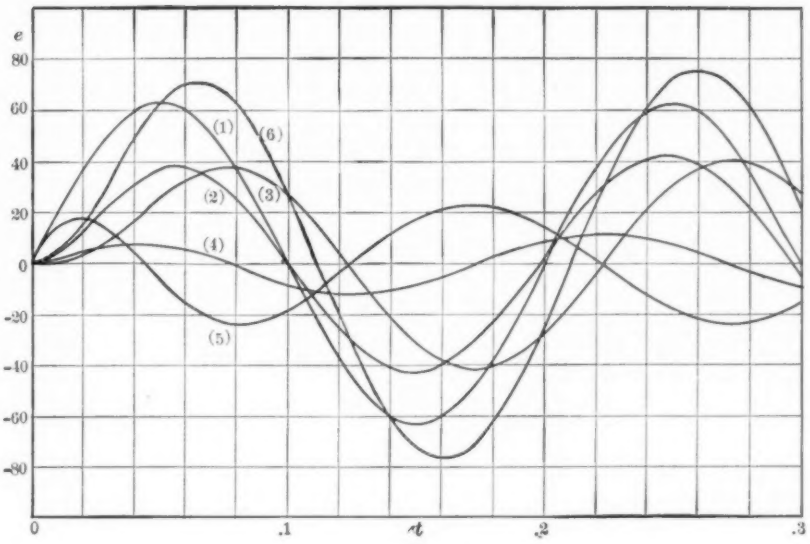


Fig. 10.

Curves of electromotive forces in the duplex cable circuits (Fig. 1), assuming that the current in the apex A is the same as that in the equivalent circuit (Fig. 2).

Curve (1). Harmonic alternating electromotive force generated by the alternator, and required to drive the current, curve (2), Fig. 4, into the apex of the cable. Same as curve (1), Fig. 4.

Curve (2). Resulting potential difference between the cable C and the earth E, Fig. 1.

Curve (2) = curve (1) — curve (3) — curve (4) — curve (5).

The cable potential curve (2) is found graphically by subtracting from the impressed electromotive force (1) the electromotive to overcome the resistance of the armature (4), the inductance of the armature (5) and the bridge condenser (3).

Curve (3). Potential drop across the bridge condenser AC, Fig. 1; $V = \frac{q}{C}$. One bridge condenser receives one-half the current and charge in the apex circuit.

Since the capacity of the bridge condenser is 80 microfarads, the equation of curve (3) is

$$\frac{q \times 10^6}{160} = -\frac{I \times 10^6}{160\omega} \cos(\omega t + \theta) + \frac{A \times 10^6}{160} \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi).$$

$$= -40.808 \cos(31.416t + .8497) - 27.224\varepsilon^{-28.87t} \sin(47.088t - 1.426).$$

Curve (4). Potential drop due to resistance of armature, equal to Ri, or $59i$. Hence

$$59i = 59I \sin(\omega t + \theta) + \frac{59A}{V\,LC} \varepsilon^{-\frac{Rt}{2L}} \sin(at + \phi + \delta).$$

$$59i = 12\ 102 \sin(31.416t + .8497) - 14.195\varepsilon^{-28.87t} \sin(47.088t - .6949).$$

Curve (5). Potential drop due to the inductance of the armature, equal to $L\frac{d^2q}{dt^2}$. same as curve (2), Fig. 8.

Curve (6). Potential drop between the apex and the earth, points A and E, Fig. 1. This may be obtained by adding curve (2) and curve (3).

$$\text{Curve } (6) = \text{curve } (2) + \text{curve } (3)$$

or, by subtracting (4) and (5) from (1),

$$\text{Curve } (6) = \text{curve } (1) - \text{curve } (4) - \text{curve } (5).$$

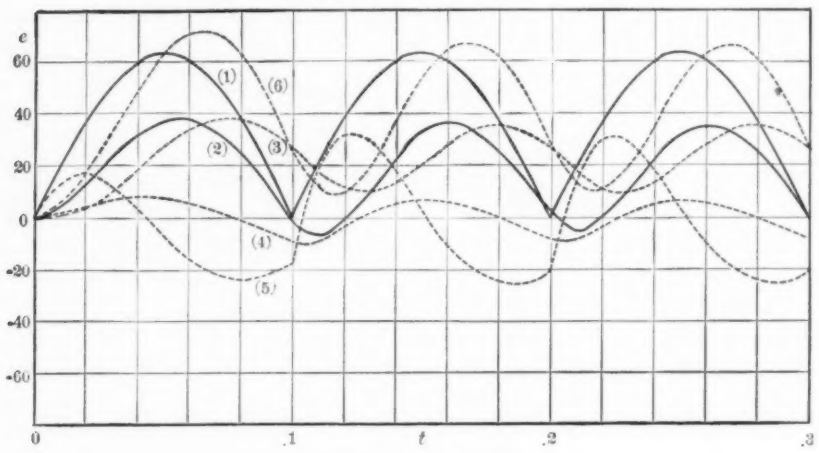


Fig. 11.

Curves of pulsating electromotive forces corresponding to those of Fig. 10. Equations of corresponding curves are the same for the first semi-cycle. For the second and succeeding semi-cycles the equations may be derived as those in Fig. 10.

THE TWO SPECIFIC HEATS OF GASES.

By W. P. Boynton.

MOST discussions and determinations of the ratio of the two specific heats of gases are based upon equations which apply strictly only to that abstraction known as the ideal gas. The external behavior of this ideal gas is given by the equation

$$pv = RT$$

where R is a constant, and T is the temperature on the absolute thermodynamical scale. Both the specific heats of the ideal gas, C_v, that at constant volume, C_p, that at constant pressure, are constant, and hence also their ratio, C_p / C_v. In actual gases both the specific heats and their ratio may and usually do vary.

The kinetic theory of gases suggests a series of values for the ratios of the specific heats, 1.667, 1.4, 1.333, 1.2857, etc., and some have affirmed that this theory could account for variations which would give values slightly less than these, but not for those slightly greater.

Some of the principal causes for variation, such as actual or approaching dissociation, will make the apparent specific heats greater, and since their difference is very nearly constant, their ratio becomes less. But some common gases give ratios slightly in excess; air and its main constituents, oxygen and nitrogen, give values averaging about 1.405, instead of 1.4.

In view of these variations I have tried to analyze the methods of measuring this ratio for a gas following van der Waals' equation,

$$(p + a / v^2)(v - b) = RT$$

which is valuable as a second approximation, coming somewhat nearer to the behavior of actual gases than does the equation for ideal gases. To restrict ourselves to the simplest form of mathe-

matical discussion, we will not simply regard a, b and R, but also C_v, the specific heat at constant volume, as constants. Then the equation of the first law of thermodynamics can be shown to take the form

$$dQ = C_v dT + \left(p + \frac{a}{v^2}\right) dv.$$

In order to find the value of C_p it is necessary to eliminate dv. Multiplying out van der Waals' equation we have

$$pv - pb + \frac{a}{v} - \frac{ab}{v^2} = RT$$

and differentiating,

$$pdv + vdp - bdp - \frac{a}{v^2} dv + \frac{2ab}{v^3} dv = RdT$$

dropping the term $\frac{2ab}{v^3} dv$ as of a smaller order than the others, collecting, and solving for dv, we have

$$dv = \frac{RdT - (v - b)dp}{p - \frac{a}{v^2}}$$

$$\left(p + \frac{a}{v^2}\right) dv = \frac{p + \frac{a}{v^2}}{p - \frac{a}{v^2}} (RdT - (v - b)dp).$$

The value of the fractional coefficient is very nearly $1 + \frac{2a}{pv^2}$ and it immediately appears that in the new equation the coefficient of dT will be

$$C_v + R\left(1 + \frac{2a}{pv^2}\right).$$

Now since the other independent variable is the pressure, p, this is what must be defined as the true specific heat at constant pressure, C_p, and the ratio of the specific heats will be

$$\frac{C_p}{C_v} = 1 + \frac{R}{C_v}\left(1 + \frac{2a}{pv^2}\right)$$

which is slightly in *excess* of the value of an ideal gas, for which $a = 0$, namely,

$$\frac{C_p}{C_v} = 1 + \frac{R}{C_v}.$$

This result is practically the same as that obtained by Boltzmann.[1]

The most easily applicable method for the experimental determination of the ratio is that of the velocity of sound. The relation of this velocity to the other properties of the gas is given by the familiar equation

$$V = \sqrt{\frac{E}{D}}$$

in which D is the density of the gas, and E the elasticity, which in this case is the ratio of the increase of pressure to the relative diminution of volume, that is

$$E = -\frac{dp}{dv/v} = -v\frac{dp}{dv}.$$

Since the sound motions are very rapid, its compressions and expansions are to be regarded as adiabatic. Now the adiabatic condition for a gas following van der Waals' equation is readily found to be given by the equation

$$\left(p + \frac{a}{v^2}\right)(v - b)^{1+\frac{R}{C_v}} = K.$$

Solving for p and differentiating,

$$p = \frac{K}{(v-b)^{1+\frac{R}{C_v}}} - a/v^2,$$

$$\frac{dp}{dv} = -\frac{\left(1 + \frac{R}{C_v}\right)K}{(v-b)^{2+\frac{R}{C_v}}} + 2a/v^3$$

$$= -\frac{\left(1 + \frac{R}{C_v}\right)\left(p + \frac{a}{v^2}\right)}{v - b} + 2a/v^3$$

[1] Gastheorie, II., p. 53.

$$E = -v\frac{dp}{dv} = \left(1 + \frac{R}{C_v}\right)\left(p + \frac{a}{v^2}\right)\frac{v}{v-b} - \frac{2a}{v^2}$$

In an absolute determination I suppose that the actually determined velocity of sound is compared with that computed by the formula

$$V = \sqrt{\frac{p}{D}}$$

which is found for an ideal gas when the compressions are assumed to be *isothermal.* This would make the deduced "ratio of the two specific heats" equal to our value of E, above given, divided by p, that is,

$$\left(1 + \frac{R}{C_v}\right)\left(1 + \frac{a}{pv^2}\right)\frac{v}{v-b} - \frac{2a}{pv^2}$$

in which the second and third factors of the first term are each greater than unity, while the second term tends to offset this increase. Performing a part of the multiplication, omitting the terms of higher orders, and grouping, we have

$$1 + \frac{R}{C_v}\left(1 + \frac{a}{pv^2} + \frac{b}{v}\right) - \frac{a}{pv^2} + \frac{b}{v}.$$

Another method consists in letting the gas previously confined at a known pressure, expand suddenly to another known pressure, noting also either the resulting instantaneous temperature, or the final pressure when the original temperature has been restored. Lummer and Pringsheim,[1] employing the former of these two methods, use the formula

$$\varkappa = \frac{\log p_0/p_1}{\log p_0/p_1 - \log \theta_0/\theta_1}.$$

For a substance following van der Waals' equation the value of $1 + R/C_v$ would be the same expression with $p + \frac{a}{v^2}$ substituted for p, or the expression given above would be equal, changing θ to T, to

$$1 + \frac{R}{C_v}\left\{1 + \frac{\log \dfrac{1 + \dfrac{a}{p_0 v_0^2}}{1 + \dfrac{a}{p_1 v_1^2}}}{\log p_0/p_1 - \log T_0/T_1}\right\}$$

[1] Winklemann, Handbuch der Physik, II., 2, p. 380.

The evaluation of the corrective term is not simple unless one has experimental data to introduce, but on the assumption that the changes of volume and pressure are small an approximation may be made by the following device; for each logarithm of a ratio may be substituted the differential of the logarithm. The expressions resulting are somewhat complicated, but dropping terms of higher orders we obtain

$$1 + \frac{R}{C_v}\left[1 + \frac{a}{pv^2}\left(1 - \frac{R}{C_v}\right)\right].$$

The three formulæ are most easily compared by taking concrete instances. Van der Waals'[1] gives values of a and b for various substances, computed by taking the atmosphere as the unit of pressure, and v as the ratio of the volume in a given state to that at 1 atmosphere and 0°C. In employing these values the three formulæ deduced thus reduce to the simple forms

$$(1) \qquad 1 + \frac{R}{C_v}(1 + 2a),$$

$$(2) \qquad 1 + \frac{R}{C_v}(1 + a + b) - a + b,$$

$$(3) \qquad 1 + \frac{R}{C_v}\left[1 + a\left(1 - \frac{R}{C_v}\right)\right].$$

The following table gives van der Waals' values of a and b for air, hydrogen and carbon dioxide, except that for hydrogen a is computed from b and the critical pressure, 15 atmospheres. The computed ratios are placed in the horizontal line with the number of the equation by which they are computed. The range of observed values is also noted.

It will be noticed that the effect of the correction term is in every case to increase the value of the ratio, except for CO_2 as computed by the second formula, where the preponderance of a makes the correction negative. These effects are of the same order of magnitude as the variations of the observed values of the ratios from the simpler values, 1.4 and 1.2857, but the experimental work is not

[1] Winklemann, Handbuch der Physik, II., 2, p. 380.

	Air.	H	CO_2
a	.002812	.0000895	.00874
b	.00197	.00047	.0023
R/C_v	.4	.4	$2/7 = .2857$
1	1.40225	1.40007	1.2907
2	1.4011	1.40057	1.2826
3	1.40067	1.400021	1.2875
Obs.			
from	1.4015	1.385	1.265
to	1.4106	1.41	1.311

sufficiently concordant to show any systematic variation depending upon the choice of method employed.

It appears that the constant a is in general the measure of the variations to be expected, or in other words, we are to anticipate the greatest variation from the simple ratios in the case of those gases or vapors which depart most decidedly from the simple gas laws on account of the mutual attractions of the molecules. It is interesting to note that the correction factors are almost identical with those computed for the coefficients of expansion, and of change of pressure, which are

$$\alpha_p = \alpha\left(1 + \frac{a}{p_0 v^2}\right),$$

$$\alpha_v = \alpha\left(1 + \frac{a}{pv^2} + \frac{a}{pv_0^2} - \frac{b}{v_0}\right)$$

where the subscript $_0$ refers to the standard conditions at 0° C.

UNIVERSITY OF CALIFORNIA,
BERKELEY, CAL., April 15, 1901.

ON A FORM OF ARTIFICIAL SUBMARINE CABLE.

By Augustus Trowbridge.

IN teaching the principles of electricity and magnetism, I have often felt the want of some form of artificial submarine cable to illustrate the effect which capacity has on the propagation of electric disturbances along a conductor.

From conversations I have had with some of my fellow physicists, I have come to the conclusion that a description of a simple piece of apparatus designed to imitate the action of a submarine cable may be of interest to teachers of physics.

From the so-called telegraph equation we have that the time which must elapse from the closing of the "home end" of a circuit possessing capacity until the potential at the far end shall have risen to a certain fraction of its final value varies as the product of the resistance and capacity of the system. Stated another way: For different lengths of the same cable the time-lags in signaling vary as the squares of the length. Of course the actual values of the time-lags depend largely on the sensibility of the galvanometer used to detect the signals.

It is customary to look upon a cable as a long Leyden jar whose inner and outer coatings are respectively the conductor and the sea-water and in which the guttapercha sheathing is the dielectric. When one end of this system is put to the pole of a battery, the other pole of which is earthed, the first element of the conductor assumes the potential of the battery, while elements further from the home end still have their original potentials. The potential at any point on the cable rises, at first very rapidly and then more slowly, until after what is theoretically an infinitely long time has elapsed a stationary state is attained when the potential at any point depends only on the resistance of the circuit and the applied electromotive force.

It is during the variable period that the capacity of the system makes itself felt and that quantity together with the resistance is what determines the time-rate of variation of the potential at any point.

The time-rate of variation of the current strength at any point may be calculated if the variation of the potential is known. In non-mathematical language the way in which the current varies may be outlined thus: The uniform current strength in the conductor after the variable period is over and the steady state has set in is determined by the electromotive force of the battery and the resistance of the cable.

During the variable period the same quantity of electricity must be in motion as in the steady state but as a great portion of this is employed at first near the home end in charging the dielectric sheathing, the current in the first elements will be far above that which will exist uniformly in the whole conductor after the steady state has set in. For this reason, the current strength at the far end will have a value far below the final value. During successive time intervals the current will fall at the home end and rise at the far end where the galvanometer will be affected as soon as the current strength has reached a value at least equal to the figure of merit of the instrument.

In order that an artificial cable shall have an appreciable time-lag, both the resistance and capacity of the system must be great.

The necessary capacity of at least several hundred microfarads is rarely, if ever, to be found in the equipment of a department of physics on account both of its bulk and expense if it is of the ordinary tin-foil type.

In designing the apparatus described in this paper it was my aim therefore to make some inexpensive substitute for the ordinary form of capacity play the same rôle in the artificial cable that the gutta-percha sheathing does in the actual cable.

I naturally endeavored to make use of the great polarization capacity which a cell consisting of dilute sulphuric acid with platinum electrodes shows when the applied electromotive force has a value less than the maximum polarization of the cell.

The first to notice the great polarization capacity of electrolytic

cells was, I believe, Mr. C. F. Varley who states in the Philosophical Transactions of the year 1872 that the capacity per square inch of platinum plates in dilute acid varies between 175 and 542 microfarads for charging electromotive forces varying between 0.02 and 1.6 volts.

In 1860 Mr. Varley constructed an artificial cable for the Atlantic Telegraph Company which wished one with which to demonstrate the feasibility of the submarine cable. I have been unable to find any description of this artificial cable but from references to it by Mr. Varley's contemporaries I infer that it was equivalent to a cable 1,200 miles long and that tin-foil capacities were used. Mr. Varley seems to have recommended the use of electrolytic condensers though as far as I can find he did not himself use them in the construction of a cable.[1]

The way in which the energy is stored up in the ordinary form of capacity and in an electrolytic capacity is, of course, very different but the effect of each in an electric circuit is the same and therefore in constructing a piece of apparatus to imitate the action of a submarine cable it is surely legitimate to use electrolytic capacity.

In a cable the capacity is distributed uniformly along the conductor while in an artificial cable we can only approximate this distribution by connecting the capacities as shown in the accompanying diagram.

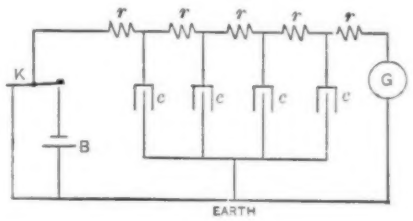


Fig. 1.

In the figure B represents a battery one pole of which is earthed, the other pole being connected to the double key K. This key serves to connect the conductor which is represented by the resis-

[1] Since writing the above I have succeeded in finding a description of Varley's artificial cable in the "Notices of the Meetings of the Royal Institution," Vol. V., 1866–1869. The inference I drew from other sources as to the nature of the capacities used appears to be correct.

tances *r*, first with the battery and then with the earth. The capacities which play the rôle of the dielectric sheathing are connected as shown. *G* is the galvanometer used to detect the arrival of a signal.

The electrolytic capacities which I used consisted of short lengths of glass tubing about 1 cm. in diameter with a platinum wire as an electrode sealed in to one end. The tube was filled nearly full of distilled water and the other electrode, which was a piece of platinum foil about 3 mm. wide, was corked in so as to dip about 5 mm. into the water.

In the cable which I have made I used 39 such cells arranged in 13 sets of 3 cells each. The 3 cells of each set were connected in parallel in the positions marked *c* in the diagram. The 13 resistances *rr* aggregated about one million ohms. The galvanometer had a figure of merit of 10^{-8} ampères which made a signal to consist of a small but very marked throw of the instrument.

It will be found convenient to mount the cells on a thin board in which a triple row of holes are drilled just large enough to allow the wire electrodes to project a few millimeters from the under side and to have this board rest on another in which are three troughs filled with mercury to serve as an earth connection for all the wire electrodes.

In this way if the board carrying the cells be lifted all the capacity is at once removed from the system and a signal is sent along the conductor with no appreciable time-lag. In other words, when the board is lifted the submarine cable becomes an overhead land line of the same length as the cable.

With the apparatus as here described the time-lag when the capacities are in, is very appreciable, being in the neighborhood of six seconds.

Of course the electromotive force of the battery used must be less than the maximum polarization of platinum electrodes in water and therefore a single Daniell or Leclanché cell is suitable.

PHYSICAL LABORATORY OF THE UNIVERSITY OF WISCONSIN.

FORCE DUE TO "CONTINUOUS IMPACT."

By Frederick Slate.

IN several well-known cases physical importance attaches to forces which are experienced as a result of some sustained process which consists of a series of impacts. It is desirable, therefore, that the idea underlying the measurement of such forces should be placed in the clearest possible connection with familiar principles of mechanics. The instances in question are usually made to depend upon the thought that force is the time-rate of momentum, the time-integral (impulse) of force being equal to the change of momentum in the line of the force. In symbols,

$$\int_{t_1}^{t} P dt = m(v - v_1); \quad P = \frac{d}{dt}(mv). \tag{1}$$

And for "steady" conditions (constant force),

$$P(t - t_1) = m(v - v_1); \quad P = \frac{m(v - v_1)}{t - t_1}. \tag{2}$$

Equation (2) is then extended to the process of (steady) continuous impact, such as that of a water-jet against a fixed surface, by associating the time-factor with m. This gives $\frac{m}{t - t_1}$ as the mass per second reaching the surface, while $(v - v_1)$ is the velocity-change produced successively in each part of the impinging stream. The ordinary formula is derived thus,

$$P = qv = a\partial v^2. \tag{3}$$

Here q denotes mass per second, ∂ density, a cross-section of jet; the impact being so arranged that the speed *in the line of the jet* is reduced to zero.

The "dimensions" of this result are right, and it is not proposed to deny the legitimateness of the argument upon which it

rests, though the reasoning may be more formally correct than convincing. For equation (1) is generally derived in connection with the continued action of force upon the same mass. The application of the equation must be somehow justified, when the permanent factor is change of speed and not mass, and the other factor represents an aggregate of differential masses; not an aggregate of differential speed-changes. The purpose of the present note is to present a supplementary point of view, which confirms the results that are ordinarily accepted on the grounds quoted above. A simplified case will suffice to make the thought plain.

Let a series of equal masses m, of the same material in the form of spheres, be dropped from a given height, at equal intervals of t seconds, upon the horizontal surface of a plate of large mass. If v_1 and v are the vertical speeds just before and just after striking, the total change in momentum within each interval t is $m(v - v_1)$. The average acceleration for the interval is then $\frac{v - v_1}{t}$, and the average force (stress) is $m\frac{v - v_1}{t}$. If now each mass m be divided into n equal parts, which are dropped at equal intervals of $\frac{t}{n}$ seconds, the average stress is, as before,

$$\frac{\frac{m}{n}\cdot\frac{v - v_1}{\frac{t}{n}}}{} = \frac{m(v - v_1)}{t} \qquad (4)$$

In other words, the time-average of the stress due to the impacts is not altered by the supposed distribution of the original mass among any number of bodies, other essential conditions remaining unchanged.

For $t = 1$, $n = 1$, and practical values of m, the curve showing stress as a function of time will consist of separate and equal sharp peaks, connected by portions of the time-axis itself which correspond to intervals of zero stress intervening between the periodic impacts. There will be violent fluctuations above and below the average value of the stress. As n is taken larger, the number of peaks for a given time increases, their height diminishes, and the departures from the average stress are reduced in magnitude. It is

possible to assign a value to n which shall make the peaked curve lie as close as is desired to the straight line representing the average stress. The final substitution of the line for the curve amounts to the adoption of a "statistical average," in place of a quantity affected with rapid and small variations, such as gas-pressure due to molecular impacts. Or we may proceed to the (mathematical) limit for *continuous* impact. Then $dm = qdt$ vanishes with dt, the change in speed for dm is produced "instantaneously," and force (the product of mass and acceleration) appears in the indeterminate form $P = 0.\infty$. On writing the limit of equation (4) we find

$$P = \lim_{n=\infty} \left[\frac{m}{n} \cdot \frac{v - v_1}{\frac{t}{n}} \right] = q(v - v_1). \qquad (5)$$

This is a steady value; *i. e.*, a *sustained average*.

It is instructive to notice the relation between this conception of the process, and the degree of approximation to steady, continuous current which depends upon the construction of a dynamo-armature in segments.

UNIVERSITY OF CALIFORNIA.

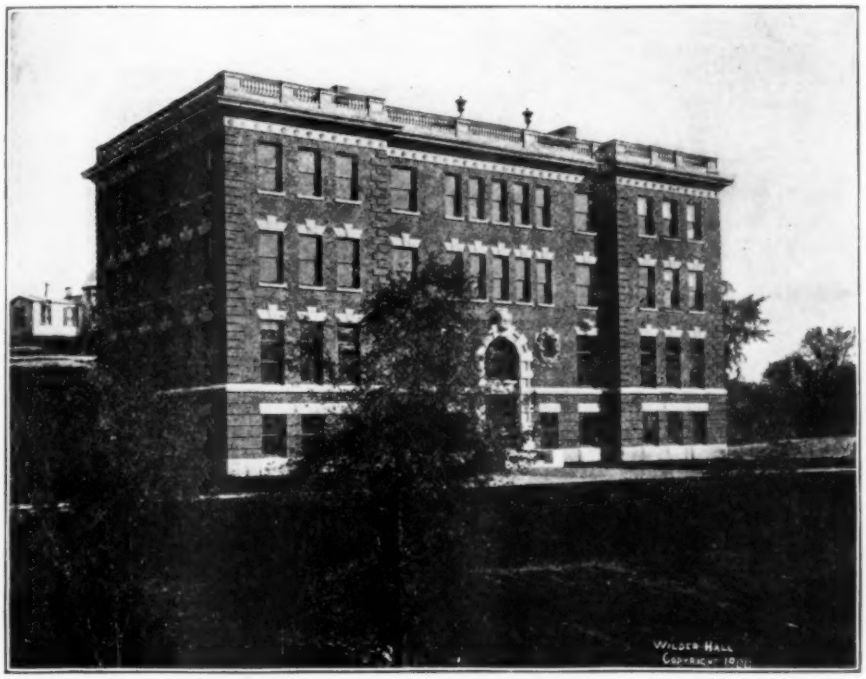
THE WILDER PHYSICAL LABORATORY OF DARTMOUTH COLLEGE.

BY E. F. NICHOLS.

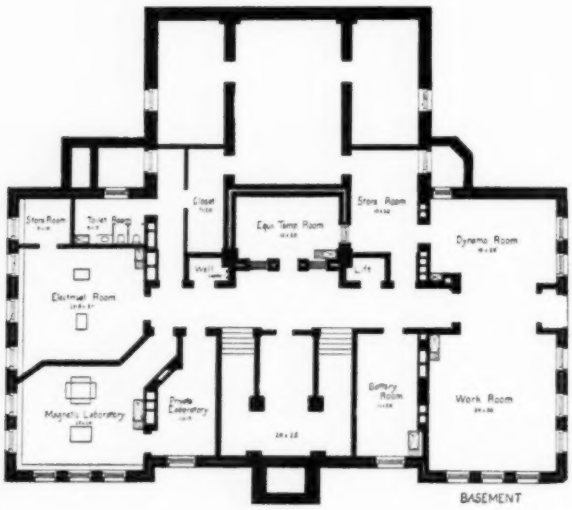
THE Wilder Physical Laboratory, the gift of the late Charles T Wilder, of Wellesley, Mass., was completed in September, 1899. The building, of which an exterior view and floor plans are reproduced here, is of brick with white granite trimmings. It has a frontage running east of north and west of south of 107 feet with a depth of 56 feet, and a rear projection 55 by 35 feet.

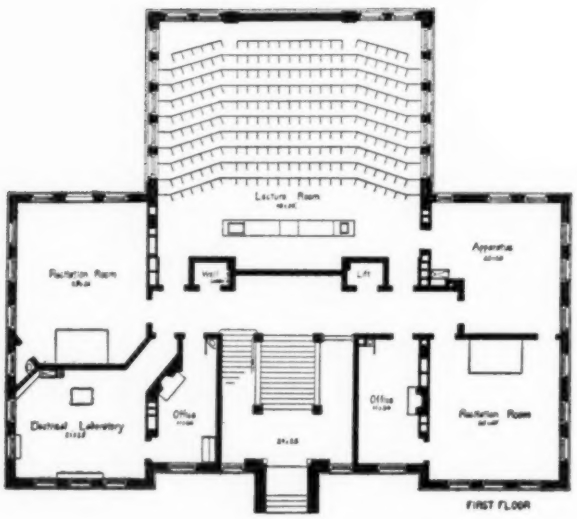
The basement section contains a workshop and dynamo room, equipped with two dynamo generators which, in common with the machine tools of the workshop, are supplied with power from a six-horse-power Pelton water wheel. The shop is provided with good metal- and wood-working lathes, and such other machine and hand tools as are necessary for the repairing and building of simple instruments. The storage battery room is stocked with 30 chloride cells (later to be increased to 60) of the type E, 5 class. These are connected in pairs to a mercury cup switchboard, permitting the grouping of the cells in a great variety of ways. This switchboard is connected to a main switchboard by three independent circuits, which makes possible a division of the battery into three sections for simultaneous use in different parts of the building. In the construction of the northwest corner of the building as high as the second floor, the use of iron has been avoided wherever possible, thus giving a practically iron-free magnetic laboratory in the basement. The basement further contains a large constant temperature room and an electrical laboratory.

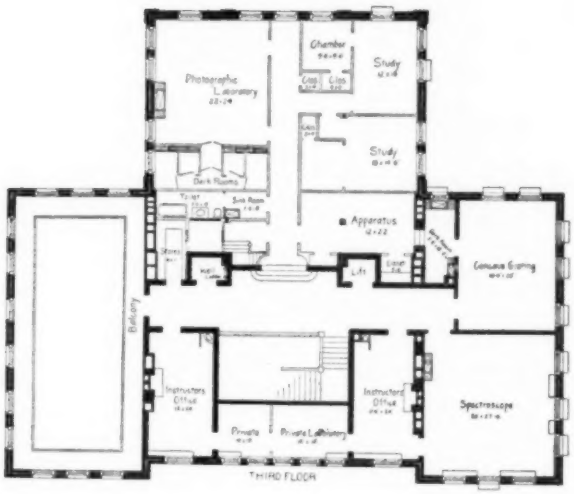
A lecture room on the first floor, seating 200, affords every convenience for the purposes of a lecture room for general physics. The windows can be darkened by shades operated by water power from the lantern table in the rear of the room. This table, not

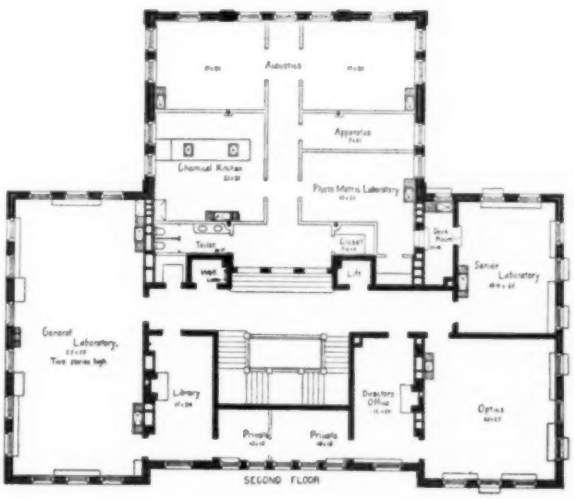


Wilder Physical Laboratory.


Store Room
Toilet Room
Closet
Equi. Temp. Room
Store Room
Dynamo Room
Well
Lift
Electrical Room
Magnetic Laboratory
Private Laboratory
Battery Room
Work Room
BASEMENT


Lecture Room
Apparatus
Recitation Room
Well
Lift
Electrical Laboratory
Office
Office
Recitation Room
FIRST FLOOR


Chamber
Study
Photographic Laboratory
Study
Dark Rooms
Sun Room
Apparatus
Lift
Convex Grating
Balcony
Instructors Office
Private
Private Laboratory
Instructors Office
Spectroscope
THIRD FLOOR


Acoustics
Apparatus
Chemical Kitchen
Photo-Metric Laboratory
Toilet
Lift
Senior Laboratory
General Laboratory
Two stories high
Library
Directors Office
Private
Private
Optics
SECOND FLOOR

shown in the plans, is provided with two power circuits to the main switchboard, with water connections and sink, illuminating gas, air under pressure, oxygen and hydrogen. The lecture table, 25 feet in length, is provided with all the conveniences of the lantern table, and steam in addition. In front of the white finished wall, used as a projection screen behind the lecture table, is a nest of movable blackboards. A roll screen is mounted on the north wall, and at an angle to it, in line with the lecture table, so that a projection lantern may be used in the front part of the room. The lecture and lantern tables, as well as all rooms in the building, are supplied with separate signal circuits, meeting in a telephone switchboard on the second floor. Thus clock, chronograph and other signals may be sent from or to any part of the building. The walls of the lecture room and those of the corridors and nearly all of the laboratory rooms are finished in dark red brick laid in tinted mortar. The room is lighted with incandescent lamps arranged between the open ceiling timbers. An apparatus and preparation room adjoins the lecture room. From a heliostat in a south window of the apparatus room a sunbeam can be projected along the lecture table.

The first floor contains, further, two recitation rooms, offices, and a research laboratory. This, like all of the other laboratory rooms, has slate shelves set in the walls at table height, has water connection, sink with slate draining boards and connections for illuminating gas and compressed air, and one or more independent power circuits to the main switchboard. The general lighting is from overhead fixtures, but for the lighting of scales or special work, wall sockets which may be plugged for portable lamps, are placed at convenient intervals.

The north end of the building, beginning with the second floor, is given up to the general laboratory. The room is open to the roof, giving great height and unusually good light. The wall space at the height of the third floor is made available for the mounting of wall apparatus by a balcony around the room.

The laboratories at the south end of the second and third floors are fitted with black shades for optical work. The southeast rooms on both floors have dark-rooms adjoining. In addition to the piping in other parts of the building, provision is here made for oxygen

and hydrogen. These rooms have additional slate shelves outside the windows for heliostat and other purposes. The corridors and doors of connecting rooms are so arranged with reference to the south windows that a beam of sunlight can be sent into many of the rooms on the north side of the building.

The portion of the second floor above the lecture room contains a photometric laboratory, a chemical laboratory, an apparatus room and two other laboratories. The rear projection on the third floor contains an apparatus room, a photographic laboratory with four developing rooms, and two lodgings for the assistants in the department.

The roof of the building is flat, adapting it for meteorological or other observations. It is accessible by both a stairway and an elevator which runs from the basement to the roof level. A neighboring shaft like the elevator well, gives a free opening 61 feet deep, suited to experiments with falling bodies for the determination of air friction, to pendulum experiments and to the measurement of elasticity and fatigue in wires.

The building is supplied with steam from the college heating station. The heating is almost entirely indirect, providing adequate ventilation. All of the rooms have direct coils which may be used in extreme weather. The lighting is by alternating current. The wiring for lighting and power circuits is enclosed in iron conduits.

The building is after plans by Mr. C. A. Rich, of New York. Credit for the carefully planned interior arrangement and details is due to Prof. Charles F. Emerson, who was assisted in parts of the work by Prof. Edwin B. Frost, Dr. A. C. Crehore and the writer.

The equipment of the laboratory is being carried forward by the income from funds left for the purpose by Mr. Wilder.

ON A METHOD FOR THE DETERMINATION OF THE CRITICAL VELOCITY OF FLUIDS.

BY H. T. BARNES AND E. G. COKER.

IT has been shown by one of us[1] that if water be heated while flowing through tubes at velocities less than the critical velocity, in which the motion is in the form of parallel lines or "stream-lines," that the distribution of heat is not uniform. In the case where the heat is applied to the outside of the tube, as in the experiments of L. Graetz,[2] only a few layers of water in direct contact with the tube will be heated, the inflow water passing directly through the center unheated. In the case where the heat is received from a central wire, the hot water flows along the wire leaving the sides of the tube unheated. It is quite otherwise for flows beyond the critical velocity, in which case the flow is eddying and sinuous, and the distribution of heat, throughout the water-column, uniform. Since the passage from direct to sinuous motion takes place suddenly, the attainment of the critical velocity may be very sharply determined by measuring the sudden change in temperature in the water-column. We first undertook to observe this for a tube heated on the outside, by measuring the increase in resistance of a platinum wire threaded through the center of the tube. Our preliminary experiments by the method of color bands showed us that the presence of the wire, at least for a tube of ½ an inch diameter and a wire 6 mils in diameter, had apparently no influence on the critical velocity. Although this method proved to be quite satisfactory, we found that it could be more simply accomplished by placing the bulb of a sensitive mercury thermometer in the path of the moving column of water just as it emerged from the tube. This had the advantage of giving us the true temperature of the water, as well as showing us the attainment of the critical velocity.

[1] Proc. Roy. Soc., 67, 238 (1900).
[2] Pogg. Ann., 18, 79 (1882).

It was a matter of considerable surprise to see the sudden jump of the thermometer thread, denoting the change in the character of the flow.

That the change in the thermometer reading indicates the point at which the steam-line motion breaks down can be shown for glass tubes by introducing the ordinary color-band test, in which case the band disappears at the same moment the jump on the thermometer take place. The appearance of flashes, occasioned by the momentary breaking down of the flow and its subsequent return to stream-line motion resulting from initial disturbances in the tank of water supplying the steady flow, are at once shown by an oscillation in the thread of the mercury thermometer, the reading going up and down with each flash. For a change in the reading of the thermometer, sufficient to observe, it is not necessary to have more than a few degrees difference in temperature between the water flowing through the tube and the walls of the tube, but in order to preserve perfectly uniform conditions, the walls of the tube must be kept at a constant temperature. This is accomplished by passing the tube through a water bath thoroughly stirred.

It would appear perhaps that there remains little to be accomplished in commencing an investigation into the question of the laws governing the flow of a liquid through a pipe—whether it should be direct or sinuous. Osborne Reynolds, Phil Trans. 1884, carried out an important investigation some years ago which treated the question both mathematically and experimentally, and gave us the laws governing the resistance of pipes for flows above and below the critical velocity. Briefly, the two methods of study adopted by him were the measurement of the point at which stream-line motion broke down, by the phenomenon of color bands, and the change from direct to sinuous motion as shown by the change in slope of the logarithmic homologues of velocity and pressure. In the first case the water entered the tube in a state free from agitation, and the breaking up of the stream-lines representing the breaking down of an unstable flow. In the second case the water entered the tube in a state of disturbance, which became direct in its passage through the tube, the arrival of the critical velocity being shown by the state of disturbance not disappearing from the flow.

A large difference was found between the values of the critical velocity measured by the two methods, which corresponds to the velocity at which stream-line motion breaks down and eddy motion changes to direct.

Owing to the advantages offered by the direct thermometric method of studying the critical velocity over the more complicated method of color bands, observations may be made on tubes other than glass. The experimental arrangements may also be made much less complicated and more suitable for fulfilling required conditions. We have already carried out some experiments to test the method on a small scale, and our preliminary results have so far shown for the sizes of tubes tried, that the critical velocity varies inversely as the diameter, according to Reynolds' well-known formula

$$v = \frac{1}{B} \cdot \frac{P}{D}$$

where B is a constant

D is the diameter

and $P = (f)\ T$.

T being the temperature centigrade.

It seems desirable to study more in detail the validity of this simple law for tubes of a greater diversity of size than Reynolds used, and more particularly for tubes of great diameter. Our experiments have not as yet been sufficiently precise to enable us to fix on the extent of the application of this formula, but as far as we have gone it is at all events approximately true. Reynolds' observations were made for only three tubes varying from 2.6 cm. diameter to .79 cm., while the effect of pressure and temperature on the critical velocity is by no means as satisfactorily known as seems desired.

Through the kind interest of Prof. H. T. Bovey, Dean of the Faculty of Applied Science, the facilities of the McGill hydraulic laboratory have been placed at our disposal, where more refined apparatus than was possible in our preliminary trials is rapidly being constructed, and where special facilities are offered for carrying out the experiments on a large scale.

MCGILL UNIVERSITY, April 27, 1901.

NOTE ON SOME MODIFICATIONS OF ELECTRIC DISCHARGES CAUSED BY DIELECTRICS.

By K. von Wesendonk.

IN view of the paper recently published by Mr. Humphreys[1] in the Physical Review, I may perhaps be permitted to draw attention to some experiments briefly described by me[2] in Wiedemann's Annalen. It is said in my paper that the effect upon an electric discharge of bodies brought near showed some peculiar differences of which, though they were surely already known, not much notice seemed to have been taken until then; and that therefore a brief description of the phenomena might not be quite useless.

In the first place, the effect of a metal plate held near the pointed pole of a spark gap, plate and pole being at the same potential, is considered. All other circumstances remaining unaltered, this effect consists chiefly in an elevation of the potential belonging to the pointed pole provided this pole be insulated. In case it has been connected to the earth so that the discharge only takes place by the influence of the other, necessarily isolated, pole, the latter will show an increase of potential. The plate brought near to the pole by changing, as may be seen, the shape of the equipotential surfaces in the vicinity of the point, reduces the potential gradient there and renders, thereby, the outflow of electricity more difficult. Thus the nature as well as the shape of the luminous discharge may be altered; for instance, instead of the electric brushes or glow discharges which alone were formerly present, one may now also observe, more or less frequently, the occurrence of sparks. As is pointed out in my paper, these experiments seem on the whole to be in favor of the Melsen's system of lightning conductors.

With dielectrics some other phenomena were observed. When a glass tube was held near to the positive pole of a spark gap,

[1] Humphreys, Physical Review, Vol. XI., page 79, 1900.
[2] Wesendonk, Wiedemann's Annalen, Vol. 49, page 296, 1893.

both terminals of which consisted of balls, then in certain positions and at suitable distances of the electrodes from each other, sparks much longer than those observed under ordinary circumstances appeared both when the positive pole was insulated and when it had been connected to the earth and the discharge went on under the influence of the insulated kathode. In neither case could any sensible change of potential at the insulated pole be discovered. The dielectric when brought near to the anode generally causes beautiful brush discharges which will then be more or less frequently transformed into sparks; but it is also possible to produce such sparks by means of the dielectric without the preceding brush discharge being seen. Quite similar is the effect of an insulated conductor brought near to the anode. In this case also the length of the sparks is considerably increased without any sensible elevation of potential. Held near to a point the dielectric, or an isolated metal piece, gave sparks but a little longer than usual, accompanied by a slight elevation of potential. Held between the poles both were efficacious only when the electrodes were very near each other. A series of small insulated pieces of metal interposed between the electrodes did not show sensibly more influence than that due to the one nearest the anode alone. In the same way water spray blown between the poles had no sensible effect.

It will be seen that the curious phenomena with which the paper of Mr. Humphreys deals have not entirely escaped former observation and that something has already been published concerning them. The explanation I believe to be the following: Since the dielectric, or the insulated piece of metal, and the nearer electrode are not at the same potential, a discharge may easily take place between the two. Now the positive discharge, as I believe to have shown, is very apt to bring down the resistance of the medium between the poles, at least partially, and so to allow a spark or a large brush discharge to take place where such cannot appear under the usual circumstances. A positively charged body is known to be very fit to give long sparks and these, I believe, are only obtained in consequence of the brush discharge which precede them.

Berlin, Jan. 30, 1901.

A SIMPLE APPARATUS FOR ILLUSTRATING FORCED VIBRATIONS.

By John R. Benton.

THE principle of forced vibrations can be illustrated by an apparatus consisting merely of a rubber band, a piece of string, and a weight.

In the adjoining figure AB represents the rubber band, hung from A; to its lower end is fastened a (non-extensible) string BC, on which the weight W is hung. If the weight is displaced vertically, and then let go, it will execute a harmonic motion, due to the elasticity of the rubber band. The period of this harmonic motion will be independent of the length of the string BC. If the weight is displaced horizontally, and then let go, it will swing like a pendulum. The period of this motion will be $2\pi\sqrt{\frac{AC}{g}}$ and can therefore be changed by changing the length of the string BC.

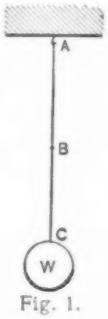


Fig. 1.

The centrifugal force of a pendulum varies periodically from 0 at the end of a swing to a maximum at the middle of a swing. If the amplitude is small, so that we may put $\theta = \sin\theta$, the centrifugal force F at any time t after passing the middle is given by the expression

$$F = mg\alpha^2 \cos^2 \sqrt{\frac{g}{l}}\, t$$

where m is the mass of the pendulum bob, α the amplitude, and l the length of the pendulum.[1]

[1] The rigorous expression for F is $F = 4mg \sin^2 \frac{\alpha}{2} \operatorname{cn} \sqrt{\frac{g}{l}}\, t$, where $\operatorname{cn} \sqrt{\frac{g}{l}}\, t$ is the cosine of the amplitude of $\sqrt{\frac{g}{l}}\, t$ in the elliptic integral $\sqrt{\frac{g}{l}}\, t = \int_0^{\phi} \frac{d\phi}{\sqrt{1 - \sin^2 \frac{\alpha}{2} \sin^2 \phi}}$.

The principle of the apparatus here described depends on the action of the variation of F in producing vibrations in the direction of the suspending string. The period of a complete cycle of variation of F is equal to half the period of the pendulum. If the weight be adjusted at such a position on the string BC that the period of the pendulum motion is twice that of the elastic vibration, and if a pendulum motion be given to the weight, then violent vibrations in the direction of the suspension will soon be set up. If however the weight is so adjusted that the period of the pendulum is equal to that of the elastic vibrations, or equal to $\frac{1}{4}$ of their period, then the pendulum swings almost as steadily as when its suspension is non-extensible.

I found it convenient to use a rubber band of cross section about 4 mm.^{2} (when not extended), and a weight of about 600 gr. The length of the pendulum when the forced vibrations attained their maximum amplitude was 40 cm.

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NEW BOOKS.

Lectures on the Lunar Theory. By JOHN COUCH ADAMS. Edited by R. A. SAMPSON. Pp. 88. Cambridge, 1900.

This set of eighteen lectures on the theory of the moon's motion is taken, without change of any sort, from the complete collection of Adams' scientific papers. The two volumes, containing these papers, were printed at Cambridge in 1896 and 1900. The first volume contained all of the published articles; the second was made up from the manuscript material left by Professor Adams, and the lectures are there published, for the first time. For those who have not means of access to thé full collection, the present reprint will be of value.

The lectures do not cover all of Adams' investigations upon the subject. They have been made up from the notes found among his papers, prepared for current use in his lectures, and changed from year to year, developing with that constant care and interest which he gave to this problem for many years.

The first lecture has been built up from the notes taken by those who had the fine opportunity of listening to the delivery, and is historical in treatment. Sufficient material was found among the manuscript papers, to supply the remaining lectures in full. Although not arranged nor completed by Adams, for any such purpose, his finished and precise method of carrying on his current work made the process of selection and combination direct and certain.

After developing the general equations for the moon's motion, the treatment followed is by separating the special problems. The Variation is first taken up, the inequality that Newton first solved. The Parallactic Inequality, the Annual Equation and the Evection are successively treated, with the resulting determination of the numerical constants. These accord well with those determined by Delauney, in his treatise upon the Theory of the Moon. The change of the Eccentricity and the motion of the Node have special treatment.

This course of lectures, it appears, did not include the evaluation of the amount of the Secular Acceleration of the moon's mean motion; though Adams had completed his investigation, and published his result in 1853, several years before the commencement of these lectures. The controversy following the publication of this constant is historical. His own contribution to the discussion, printed in 1860, is a model of earnest, vigorous and at the same time courteous statement.

Nor are any of the results from his investigation of the Lunar Parallax given here. Aside from these lectures, fully a quarter of Adams' published articles were upon subjects connected with the Lunar Theory.

The lectures close with a discussion of Hills' method of treating portions of the same problem. In some respects the two great mathematicians had been working on parallel lines. While the details of the processes employed differ, the methods and results confirm each the other.

R. H. TUCKER.

Atmospheric Radiation. By FRANK W. VERY. United States Weaher Bureau. (Bulletin G of the Department of Agriculture.) Washington, 1900. Pp. 134.

In this important memoir Professor Very presents the results of an extended investigation of the very difficult subject of radiation from the various components of the atmosphere. The work includes, from the nature of the case, the study, separately, of radiation at various temperatures and pressures, from dry air, aqueous vapor and carbon dioxide.

The problem of atmospheric radiation is greatly complicated by the fact that both moisture and carbon dioxide possess great absorbing power for the particular wave-lengths which they are capable of radiating so that they become opaque to their own radiation even in layers of moderate thickness. The opacity of gases is shown to depend upon the selective character of their radiation, so that those gases which are powerful radiants of certain wave-lengths, in thin layers become more rapidly opaque to their own radiation than poorer radiators, such as dry air. In the case of the two constituents just mentioned, namely, water and carbon dioxide, it was found for example that the radiation with increasing depth of layer rapidly approaches a maximum so that a five-foot layer of the latter gas radiates little more than a two-foot layer and no more than a three-foot layer; while dry air radiates very nearly in proportion to the depth. The extraordinary fact previously noted by Paschen that vapors such as ammonia, carbon dioxide, etc., have in the regions of maximum radiating power approximately equal to that of a black body, is abundantly confirmed. Various important applications of the results of this research to the complex problems of meteorology are treated in the closing pages of the memoir.

To the physicist Professor Very's work is of very great direct interest, not only for the results which he has obtained but as an admirable and inspiring example of the thorough-going, exhaustive and successful experimental research upon an intricate problem.

E. L. N.

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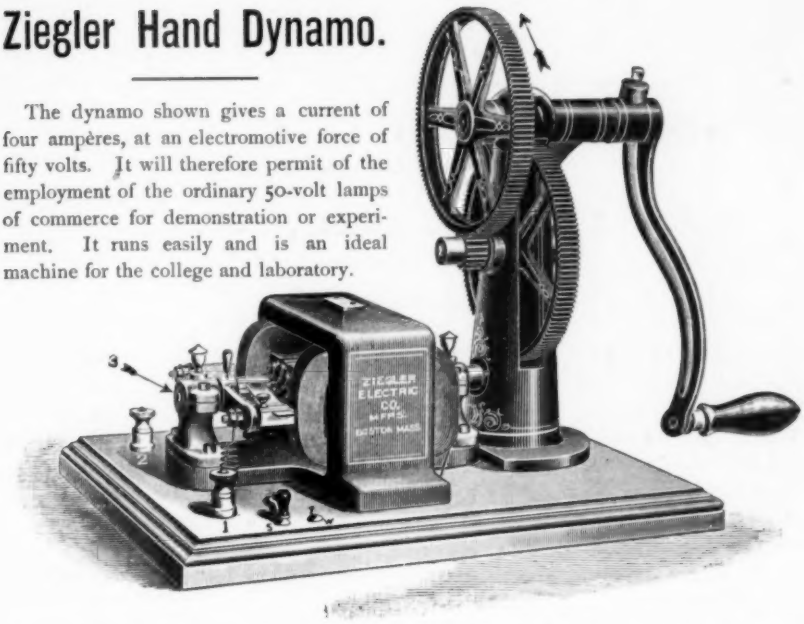

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